A New Model to Evaluate the Macroeconomic Impact of Regional Economic and Policy Shocks in Belgium

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Abstract

This paper presents the structure and the main equations of the last version of PREVIEW, a small calibrated bottom up macroeconomic model being developed at IWEPS to evaluate the impact of regional economic and policy shocks on the three main regional economies of Belgium.

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Contents

1 Introduction

This paper presents the structure and the main equations of a *bottom up* macroeconomic model mainly aimed at evaluating the impact of aggregated public policies decided at the regional level. The model could also be used to assess the impact of federal aggregated policies as well as the impact of regional or international economic shocks. It is being developed in the context of the sixth State reform recently implemented in Belgium[1]. The main features of this reform concern the transfer of a large block of competences from the federal level to the federated entities, on the one hand, and a major revision of the Special Finance Act (SFA), on the other hand. Specifically, three important fields of competence have been transferred to the regions: (i) employment policy (partially, including policy measures for target groups), (ii) mobility and road safety and (iii) tax reductions and tax credits. This reform tends to increase the fiscal autonomy of regions while maintaining a solidarity between the federated entities to ensure the long term sustainability of national public finances.

There are surprisingly few simulation tools available in Belgium to assess the macroeconomic and budgetary effects of policy changes that will now be taken at a lower level of governance. The most accomplished one is probably HERMREG, a regional macroeconometric model closely connected to the aggregated national model HERMES, the main goal of which is to provide regional medium term economic projections. However this model has not yet a full bottom up structure that would allow to properly identify the propagation effects of regional policies inside and outside the regions[2]. Such a tool is all the more important as Belgian regions exhibit significant differences in terms of growth performance, unemployment rate and structure of population while being highly connected in terms of product trade flows and labour mobility.

It is specifically to fill the gap that we are developing a model named PREVIEW, an acronym for "**P**olicy and **R**egions: **EV**aluating the **I**mpact on the **E**conomy of **W**allonia"[3]. Althought the model focuses above all on Wallonia, it also allows to assess the effects of Walloon policies on the two remaining Belgian regions as well as the ones of non local regional policies on Wallonia.

[1] For the most important institutional evolutions of Belgian federalism stemming from the implementation of the sixth State reform, see http://www.on-federalism.eu/attachments/213_download.pdf

[2] Note that significant efforts have been done recently concerning the bottom up structure of HERMREG.

[3] Wallonia is one of the three Belgian regions at the NUTS 1 level.

The initial proposal of the project was favouring the theoretical development of a small general equilibrium macroeconomic model with a *bottom up* structure, inspired by the Keynesian school of thought, but also incorporating elements from geographical economy. Key elements can be summarized as follows. Firms and households are specific to regions and behave as optimizing agents. The first ones minimize their production costs while the second ones maximize a mixed utility of consumption and leisure time. Constraints relate respectively to a production function reacting to demand conditions and to a budget line that controls the households consumption. Regional production is perfectly mobile and can meet any demand expressed by any region. Labour force is heterogenous and perfectly mobile, and the regional distribution of workers is explained by differentials observed both in time spent to commute and in wages net of travel costs.

Unfortunately, as the research was ongoing, we faced some technical difficulties when translating the intended architecture of the model into equations. This notably concerned the application of the utility approach based on consumption and leisure time of households in the context of interconnected regions. To save time and end up more rapidly with an operational version of the model we had to turn to a more standard way of macromodelling. It is important to keep in mind that the aim of the PREVIEW project is not to test whether an enriched theoretical setting is validated (or not) by regional statistics, but rather to propose a flexible and tractable tool allowing the evaluation of the impact of alternative modulation in the policy mix implemented by regional authorities. This may be viewed as the search for a trade-off between the theoretical and the applied research.

The actual version of the model is thus based on a simplified version assuming a fixed interregional structure both for regional labour markets and for regional product markets. The key element is the *bottom up* structure of the model consisting in three regions and two populations of workers, one of which is considered to be vulnerable (and as such a potential group for targeted policies). In the last version of the model, workforce is split according to the education level.

We believe that our flexible model could easily be extended, within a reasonable period of time, so as to be able to perform simulations of the impact of a wide range of economic and policy shocks affecting regions, providing regional policy makers with a useful tool to conduct effective and efficient macroeconomic policies.

The sequel of the paper elaborates on the main modules of the model.

2 The core of PREVIEW

Let us start this section with a very brief description of the model. PREVIEW is a (mainly) static macroeconomic model in which regional economies are essentially driven by demand. A certain level of *ad hoc* dynamics is introduced with a cost-price-wage loop and the standard capital accumulation. The model has a *bottom up*, or aggregative, structure meaning that reaction functions of economic agents are set up at the regional level, and decisions are then aggregated to form the national level. Economic agents consist in firms and households, and public policies are designed both by the federal State and by federated entities (i.e. the regional authorities). Product and labour markets are specific to regions and are in close interplay. The money market is confined to an exogenous national interest rate. There is currently no sectoral breakdown in the economy but the regional workforce is divided in two groups of qualification : a low level of education, i.e. at most lower secondary education, versus a high level of education. This distinction is important as target group labour policies, recently became a regional competence, are mostly addressing the less educated category.

We now turn to a more detailed description of the model.

2.1 Regional product markets and GDP

Supply side. Regional employers ajust their production to satisfy the demand addressed to their products. In such a context they are supposed to minimize the total cost of production subject to a fixed production function. The output technology is decribed by a Cobb Douglas function with constant returns to scale and with subtitutable labour and capital inputs:

$$VA_i = \widehat{A}_i e^{\widehat{\varphi}_i trend} \left(L_i^{LQ}\right)^{\widehat{\beta}_i^{LQ}} \left(L_i^{HQ}\right)^{\widehat{\beta}_i^{HQ}} (K_i)^{1-\widehat{\beta}_i^{LQ}-\widehat{\beta}_i^{HQ}} \tag{1}$$

with VA_i the value added of (firms located in) region i, L_i^q the number of workers in region i with a low level of qualification ($q = LQ$) versus a high level of qualification ($q = HQ$), respectively, and K_i the stock of capital used by firms i.

Parameters β_i^q relate to the regional elasticity of output to labour of category q, while φ_i is a region specific trend productivity coefficient and A_i a region specific scale factor. Note that in the sequel of the paper parameters to be estimated are systematically decorated with the hat operator.

Total cost of production sums up compensation of employees and the total cost of capital:

$$COST_i = \omega_i^{LQ} L_i^{LQ} + \omega_i^{HQ} L_i^{HQ} + c_i K_i \tag{2}$$

with ω_i^q the average nomimal wage cost for category q observed in region i, and c_i the user cost of regional stock of capital.

The cost of labour input q gets the following definiton:

$$\omega_i^q = (1 + \widetilde{\tau}_i^{E,q}) w_i^q \tag{3}$$

with $\widetilde{t}_i^{E,q}$ the implicit employers' social security (ESS) contribution rate, considered to be exogenous and hence decorated with the tilde operator, and w_i the nominal gross wage rate proposed by firms of region i. The key point is that, in Belgium, a part of $\widetilde{\tau}_i^{E,q}$ is in the hands of federal State (mandatory ESS contribution rate and structural ESS reduction rate) and the remaining is (mostly) in the hands of federated entities (target group ESS reduction rate).

The user cost of capital input is defined as:

$$c_i = \left(\widetilde{r} + \widehat{\delta}\right) \widetilde{p}_i^I \tag{4}$$

depending on the exogenous national interest rate r, on the fixed rate of capital depreciation δ (identical whatever the region) and on the exogenous regional unit investment price $\widetilde{p}_i^I$.

Demand side. On the one hand final, demand for goods and services expressed by regional agents is composed of different aggregates: household final consumption (C), investment (I) and exports (X) by firms, and exogenous public expenditures ($\widetilde{G}$). On the other hand, regional firms also consume production goods that are destroyed during the production process, and this intermediate demand is noted Z. Functional forms for endogenous C, I, X and Z will be presented later.

Regional structure of production and GDP. Supply and demand sides of regional product markets are interconnected and product flows are modelled according to the same architecture as the one used in regional input output (RIO) analysis. The notations are summarized in table 1. Index i refers to region i as a line of the table while index j refers to region j as a column of the table.

Table 1 : regional input output architecture

$\vdots$ $\cdots \overline{Z}^{Q}_{i,j} \cdots$ $\vdots$	$\vdots$ $\cdots \overline{C}^{Q}_{i,j} \cdots$ $\vdots$	$\vdots$ $\cdots \overline{G}^{Q}_{i,j} \cdots$ $\vdots$	$\vdots$ $\cdots \overline{I}^{Q}_{i,j} \cdots$ $\vdots$	$\vdots$ $\cdots \overline{X}^{Q}_{i,j} \cdots$ $\vdots$	$\vdots$ $\overline{Q}_i$ $\vdots$
$\cdots \overline{Z}^{Q}_{j} \cdots$ $\cdots \overline{Z}^{M}_{j} \cdots$ $\cdots \overline{Z}_{j} \cdots$ $\vdots$ $\cdots Z_j \cdots$	$\cdots \overline{C}^{Q}_{j} \cdots$ $\cdots \overline{C}^{M}_{j} \cdots$ $\cdots \overline{C}_{j} \cdots$ $\vdots$ $\cdots C_j \cdots$	$\cdots \overline{G}^{Q}_{j} \cdots$ $\cdots \overline{G}^{M}_{j} \cdots$ $\cdots \overline{G}_{j} \cdots$ $\vdots$ $\cdots G_j \cdots$	$\cdots \overline{I}^{Q}_{j} \cdots$ $\cdots \overline{I}^{M}_{j} \cdots$ $\cdots \overline{I}_{j} \cdots$ $\vdots$ $\cdots I_j \cdots$	$\cdots \overline{X}^{Q}_{j} \cdots$ $\cdots \overline{X}^{M}_{j} \cdots$ $\cdots \overline{X}_{j} \cdots$ $\vdots$ $\cdots X_j \cdots$	
$\cdots VA_j \cdots$ $\cdots \overline{Q}_j \cdots$					
$\cdots PIB_j \cdots$ $\cdots M_j \cdots$					

Noting Q_i the production of region i, we first pinpoint the horizontal identity:

$$\overline{Q}_i = \sum_{j=1} \left(\overline{Z}^{Q}_{i,j} + \overline{C}^{Q}_{i,j} + \overline{I}^{Q}_{i,j} + \overline{\overline{G}}^{Q}_{i,j} + \overline{X}^{Q}_{i,j} \right) \tag{5}$$

where agregate $\overline{B}^{Q}_{ij}$, $B = Z, C, I, G, X$, is the part of regional production $\overline{Q}_i$ delivered to demand of type B expressed by agents located in region j, or equivalently, the part of demand of type B expressed by agents located in region j which is satisfied by a production coming from region i. All variables are decorated with the bar operator as they are valued in basic prices.

We next consider international imports by regional firms. Again we align with the RIO structure so that:

$$\overline{M}_j = \overline{Z}^{M}_{j} + \overline{C}^{M}_{j} + \overline{G}^{M}_{j} + \overline{I}^{M}_{j} + \overline{X}^{M}_{j} \tag{6}$$

with $\overline{B}^{M}_{j}$ the part of international imports of firms located in region j imputed to demand of type B expressed by agents located in region j. Actually imports by region j can only be delivered to agents located in the same region.

We do not go further in the details of the input output module of the model but bring light to five key elements related to the vertical structure of (5). It concerns: (1) the fixed technical coefficients of the production $\widehat{g}^{Q}_{i,j} = \overline{Z}^{Q}_{i,j} / \overline{Q}_j$, (2) the fixed technical coefficients of imports $\widehat{g}^{M}_{j} = \overline{Z}^{M}_{j} / \overline{Q}_j$, (3) the fixed production structure of the final demand $\widehat{h}^{B}_{i,j} = \overline{B}^{Q}_{i,j} / \overline{B}^{Q}_{j}$, with

$\overline{B}_j^Q = \sum_i \overline{B}_{i,j}^Q$, (4) the fixed production and import structure of the final demand in $\overline{B}_j = \overline{B}_j^Q + \overline{B}_j^M$, and (5) the connection between basic / c.i.f.[4] prices and purchaser prices which include net taxes on products, so that $B_j = \left(1 + \widehat{t}_j^B\right) \overline{B}_j$.

In the end, such a RIO framework allows us to express gross domestic product at market prices (GDP) and imports at purchase prices of any region as a linear combination of the final demand aggregates observed at the regional level:

$$GDP_i = \sum_j \left(\widehat{\alpha}_{i,j}^{GDP,C} C_j + \widehat{\alpha}_{i,j}^{GDP,I} I_j + \widehat{\alpha}_{i,j}^{GDP,G} \widetilde{G}_j + \widehat{\alpha}_{i,j}^{GDP,X} X_j \right) \tag{7}$$

$$M_i = \sum_j \left(\widehat{\alpha}_{i,j}^{M,C} C_j + \widehat{\alpha}_{i,j}^{M,I} I_j + \widehat{\alpha}_{i,j}^{M,G} \widetilde{G}_j + \widehat{\alpha}_{i,j}^{M,X} X_j \right) \tag{8}$$

This is the core of the driven-by-demand feature of model PREVIEW. The parameters of equations (11) up to (8) are mathematically related to the set of fixed coefficients characterizing the five vertical relations that have just been highlighted. Regional trade also rests on this RIO architecture, see section 2.6.

By construction:

$$VA_i = \frac{GDP_i}{1 + \tau_i^{TAX}} \tag{9}$$

$$Z_i = Z_i \left[VA_i \right] \tag{10}$$

$$Q_i = VA_i + Z_i \tag{11}$$

2.2 Employment and unemployment

Workers (employees). Minimization of the total production cost (2) with respect to the production technology (1) leads to the following labour demand equations for the two categories of workers occupied in region i:

$$\frac{L_i^{LQ}}{VA_i} = \zeta_i \left(\frac{\omega_i^{HQ}}{\omega_i^{LQ}} \right)^{\widehat{\beta}_i^{HQ}} \left(\frac{c_i}{\omega_i^{LQ}} \right)^{1 - \widehat{\beta}_i^{LQ} - \widehat{\beta}_i^{HQ}} \tag{12}$$

[4] c.i.f. stands for cost, insurance and freight, and concerns the valuation of international imports.

and

$$\frac{L_i^{HQ}}{VA_i} = \zeta_i \left(\frac{\widehat{\beta}_i^{LQ}}{\widehat{\beta}_i^{HQ}}\right)^{-1} \left(\frac{\omega_i^{HQ}}{\omega_i^{LQ}}\right)^{\widehat{\beta}_i^{HQ}-1} \left(\frac{c_i}{\omega_i^{LQ}}\right)^{1-\widehat{\beta}_i^{LQ}-\widehat{\beta}_i^{HQ}} \tag{13}$$

with

$$\zeta_i = \left(\widehat{A}_i e^{\widehat{\varphi}_i TREND}\right)^{-1} \left(\frac{\widehat{\beta}_i^{LQ}}{\widehat{\beta}_i^{HQ}}\right)^{\widehat{\beta}^{HQ}} \left(\frac{\widehat{\beta}_i^{HQ}}{1-\widehat{\beta}_i^{LQ}-\widehat{\beta}_i^{HQ}}\right)^{1-\widehat{\beta}_i^{LQ}-\widehat{\beta}_i^{HQ}} \tag{14}$$

Self-employment. Self-employment in region i is assumed to be related to the numbers of employees in region i, on the one hand, and to GDP of region i, on the other hand:

$$LI_i = LI\left\{L_i, GDP_i; \widehat{\lambda}_q\right\} \tag{15}$$

with $\widehat{\lambda}_i$ a set of fixed parameters (the functional form ant its properties will be presented at the October 23 meeting of the Scientific Support Committee of the project).

This broadly reflects the coexistence of 'push' and 'pull' factors, or to a lesser extent, the divergence between the necessity entrepreneur and the opportunity entrepreneur[5].

Then, regional LI is allocated according to the education level, with a fixed disaggregation key.

Regional working population. We define LR_i^q the employed population of region i with qualification level q, i.e. the number of residents of category q who are occupied in the labour market, as:

$$LR_i^q = \sum_j \widehat{\pi}_{i,j}^q L_j^q \tag{16}$$

where $\widehat{\pi}_{i,j}^q$ is the (fixed) share of labour of category q assigned by regional firms j to residents of region i. Note that non Belgian labour market is also considered as a whole in a Euro Zone region.

Total working population of region i can thus be written by category:

$$WP_i^q = LR_i^q + LI_i^q \tag{17}$$

[5]See a.o. Dawson et al. (2009).

Unemployment. Workforce available in region i is noted WF_i and is assumed to be exogenous. Therefore regional unemployment level and regional employment rate for category q are defined respectively as:

$$U_i^q = \widetilde{WF}_i^q - WP_i^q \tag{18}$$

and

$$u_i^q = 1 - \frac{WP_i^q}{\widetilde{WF}_i^q} \tag{19}$$

2.3 Stock of capital and investment

The minimizing cost strategy leads to the following solution for the demand of capital by firms located in region i:

$$\frac{K_i}{VA_i} = \zeta_i \left(\frac{\widehat{\beta}_i^{LQ}}{1 - \widehat{\beta}_i^{LQ} - \widehat{\beta}_i^{HQ}} \right) \left(\frac{\omega_i^{HQ}}{\omega_i^{LQ}} \right)^{\widehat{\beta}_i^{HQ}} \left(\frac{c_i}{\omega_i^{LQ}} \right)^{-\widehat{\beta}_i^{LQ} - \widehat{\beta}_i^{HQ}} \tag{20}$$

Investment is obtained using the dynamic capital accumulation equation:

$$I_i = K_i - (1 - \widehat{\delta}) K_i[-1] \tag{21}$$

2.4 From incomes to consumption

Primary incomes and household disposable income. We first define total labour incomes RL_i as labour incomes earned by residents of region i (WB_i) plus mixed income by regional self-employed workers (RM_i):

$$\begin{aligned} RL_i^q &= WB_i^q + RM_i^q \\ &= \sum_j \widehat{\pi}_{i,j}^q w_j^q L_j^q + \widetilde{w}_i^{I,q} LI_i^q \end{aligned} \tag{22}$$

with $w_i^{I,q}$ the exogenous mixed income per head observed in region i.

A specific and exogenous average implicit personal social security (PSS) contribution rate $\widetilde{\tau}_i^{P,q}$ is then taken from regional total labour incomes. The remaining is added to non labour revenues of residents, consisting in (both exogenous) property incomes ($\widetilde{RP}_i^q$) and gross operating surplus of households ($\widetilde{GOSM}_i^q$), to form the before tax income of residents of region i:

$$BTI_i^q = \left(1 - \widetilde{\tau}_i^{P,q}\right) RL_i^q + \widetilde{RP}_i^q + \widetilde{GOSM}_i^q \tag{23}$$

After deduction of income taxes based on $\tau_i^{I,q}$, the exogenous regional global implicit tax rate on incomes, and addition of unemployment allowances and other exogenous net transfers by public sector to households (OTM_i), we end up with the regional level of household disposable income HDI_i:

$$HDI_i^q = (1 - \widetilde{\tau}_i^{I,q})BTI_i^q + w_i^{U,q}U_i^q + \widetilde{OTM}_i^q \tag{24}$$

with $w_i^{U,q}$ the average unemployment benefit for unemployed with qualification q living in region i.

Consumption. Real private consumption of regional households is defined as a fixed share $\widehat{\theta}_i^q$ of their disposable income expressed in real value:

$$C_i^q = \widehat{\theta}_i^q \frac{HDI_i^q}{p^C} \tag{25}$$

with p^C the consumption price index (see next section).

2.5 Inflation, wages and profits

An important feature of the model relies on its cost-price-wage loop which, besides, concerns both regional and national levels.

Total cost dynamics. Growth of the average production cost supported by regional firms (i.e. the inflation in the price of value added) is assumed to be linearly related to the growth of average regional wage costs and to the growth of the regional user cost of capital:

$$\Delta \ln p_i^{VA} = \widehat{\beta}_i^{LQ} \Delta \ln \omega_i^{LQ} + \widehat{\beta}_i^{HQ} \Delta \ln \omega_i^{HQ} + \widehat{\beta}_i^{K} \Delta \ln c_i \tag{26}$$

where, recalling the constant return-to-scale assumption, $\widehat{\beta}_i^{LQ} + \widehat{\beta}_i^{HQ} + \widehat{\beta}_i^{K} = 1$

Inflation. Consumer price index (CPI) p^C is only observed at the national level. We retain the assumption of a constant markup applied by Belgian firms whatever the region of location, so that national CPI evolves at the same pace as the national average of regional production costs:

$$\Delta \ln p^C = \Delta \ln p^{VA} \tag{27}$$

with:

$$p^{VA} = \frac{\sum_i p_i^{VA} VA_i}{\sum_i VA_i} \tag{28}$$

Wage dynamics. Price indexation of wages is automatic in Belgium. Beyond that rule, we relate the growth of regional real wage rate and the local unemployment rate, recognizing the existence of regional Phillips curves by category of workers. Also, we take into account the existence of an exogenous upper limit in the growth in nominal average wages after deduction of inflation and scale increases (i.e. the wage moderation). Besides the effect of a jump in wage indexation can be tested by setting a dummy variable SI to 1.

This leads to equation:

$$\Delta \ln w_i^q = \widehat{\varepsilon}_{0,i}^q + (1 - SI)\, \Delta \ln p^C + Min \left[\frac{\widehat{\varepsilon}_{1,i}^q}{u_i^q}; \widetilde{NORM} \right] \tag{29}$$

with parameter $\widehat{\varepsilon}_{0,i}^q$ reflecting the scale increase in the wage rate earned by worker of category q occupied in region i.

When the degree of wage moderation allows it, equation (29) captures the influence of trade unions on the wage setting process but also, indirectly, the impact of a demand shift on wages.

Unemployment allowances. Average unemployment benefit w_i^U depends on the level of qualification and is region-specific. It is automatically revised with inflation so that:

$$\Delta \ln w_i^{U,q} = \Delta \ln p^C \tag{30}$$

Profits. Nominal profits made by regional firms are defined as nominal value added minus compensation of employees :

$$PROFIT_i = p_i^{VA} VA_i - COST_i \tag{31}$$

2.6 International and regional trade

The model allows to calculate external trade accounts for each region that distinguish between international flows of products and regional flows of products.

International trade. International exports of regional firms X_i are expected to depend positively on the global GDP of the main partners of the Belgian regions, taken here as the Euro Zone (excluding Belgium), and also on a price competitiveness indicator. This indicator is defined as the ratio of the nominal unit labor cost (UCL) observed in the Euro Zone, considered to be exogenous, and the nominal UCL observed in region i:

$$X_i = X_i \left\{ \widetilde{GDP}_{EZ}, \frac{\widetilde{ULC}_{EZ}}{ULC_i}; \widehat{\xi}_i \right\} \tag{32}$$

with $\widehat{\xi}_i$ a set of fixed parameters (actually, we have selected a log-linear form for the above export equations) and:

$$ULC_i = \left(\frac{\omega_i^{LQ} L_i^{LQ} + \omega_i^{HQ} L_i^{HQ}}{L_i^{LQ} + L_i^{HQ}} \right) / \left(\frac{VA_i}{L_i^{LQ} + L_i^{HQ} + LI_i^{LQ} + LI_i^{HQ}} \right) \tag{33}$$

International imports of firms i have already been defined in the section dedicated to the RIO structure of the model, see equation (8).

Regional trade. Trade among the Belgian regions is also formalized using the RIO approach developed in section 2.1. Using the same notations, exports from region i to region j are easily checked to have the following expression:

$$XR_{i,j} = Z_{i,j}^Q + C_{i,j}^Q + G_{i,j}^Q + I_{i,j}^Q + X_{i,j}^Q, \quad i \neq j \tag{34}$$

while, by construction, regional imports of region i from region j are defined as:

$$MR_{i,j} = XR_{j,i} \tag{35}$$

Note that all variables in the last two equations are valued in purchase prices. In their final expressions, the XRs can be matematically expressed as a linear combination of the regional production and the different regional final demands, the parameters of which are related to the fixed coefficients of the RIO structure.

2.7 Public finance

At the current stage of development the model contains a small public finance module that captures, at the regional level, **four categories of public revenue**, namely employer's and personal social security contributions by

level of qualification, income taxes by level of education, and net taxes on products (with tax rates coming from the RIO table):

$$ESSC_i^q = \widetilde{\tau}_i^{E,q} w_i^q L_i^q \tag{36}$$

$$PSSC_i^q = \widetilde{t}_i^{P,q} RL_i^q \tag{37}$$

$$IT_i^q = \widetilde{\tau}_i^{I,q} BTI_i^q \tag{38}$$

$$TAX_i = \tau_i^{TAX} VA_i \tag{39}$$

The **three categories of public expenditure** observed at the regional level relate respectively to government spendings, other net transfers to households and unemployment allowances, which add up to:

$$\widetilde{G}_i + \sum_q \widetilde{OTM}_i^q + \sum_q w_i^{U,q} U_i^q \tag{40}$$