Are hiring subsidies detrimental to employment stability? Insights from a calibrated matching model*

Matthieu Delpierre†

March 14, 2019

Abstract

This paper aims at exploring employers' incentives to hire and dismiss temporarily subsidized workers in a matching model with job heterogeneity. We analyze transitions from temporary to permanent employment, which allows us to study unemployment but also employment composition. The analytical model concludes that hiring subsidies lead to a higher turnover and a larger mass and proportion of temporary jobs, but is inconclusive regarding their impact on unemployment and the mass of permanent jobs. The mechanism whereby separations are stimulated by hiring subsidies originates from job heterogeneity. Higher type jobs are on average more productive and are thus more likely to survive beyond the subsidized period. We show that hiring subsidies decrease the profitability threshold, with lower type vacancies entering the labor market. Congestion externalities increase and all vacancies are less easily filled, including high type vacancies. A crowding out effect, by which high type vacancies would be fewer, is also a theoretical possibility. The model is calibrated with data from the Southern part of Belgium (Wallonia). Simulations reveal that increasing hiring subsidies at the margin would further decrease unemployment, but this effect would be very limited and net job creations would almost entirely consist of temporary jobs. Our results also indicate that if the subsidy were set too high, then permanent employment would be negatively affected and, to a lower extent, aggregate employment.

*I am grateful to Bruno Van der Linden, Frédéric Gaspart, Frédéric Verschueren, Valérie Vander Stricht and Laurence Vanden Dooren for useful comments and suggestions.

†Walloon Institute of Evaluation, Foresight and Statistics (IWEPS), Belgium, m.delpierre@iweps.be

1 Introduction

Targeted hiring subsidies are widely used as a tool to alleviate unemployment among vulnerable workers and to improve individual job market trajectories. Owing to their temporary nature, hiring subsidies are often considered as a cheaper alternative to wage subsidies to achieve this twofold policy objective. The reason thereof is that they only apply to the flow of workers out of unemployment, not to the whole stock of workers, and hence mechanically entail lower deadweight losses. However, while their positive impact on job creations is generally acknowledged (Blundell et al. (2004), Cockx et al. (2004), Kangasharju (2007), Cahuc et al. (2014), Sjogren & Vikstrom (2015)), their effect on separations is more controversial and less studied: more job destructions may indeed get induced either among the non-beneficiaries, through a substitution effect; or among former beneficiaries, as employers, who are facing the incentive to make new hires, may well leave their aggregate labor demand unchanged and thus dismiss workers with similar characteristics (Dahlberg & Forslund (2005)). This latter effect, which is known as the displacement effect, tends to cause a higher turnover and might adversely affect the long term prospects of the targeted unemployed.

This paper aims at exploring employers' incentives to hire and dismiss temporarily subsidized workers in a matching model with job heterogeneity. We focus on the targeted segment of the labor market and explicitly consider transitions from temporary to permanent employment. This allows us to study unemployment, but also individual trajectories and the respective shares of temporary and permanent jobs in total employment. The analytical model concludes that hiring subsidies lead to a higher turnover and a larger mass and proportion of temporary jobs, but is inconclusive regarding their impact on unemployment and the mass of permanent jobs. The ambiguous effect on unemployment is not new as it appears in Mortensen & Pissarides (2001). However, as discussed below, the adverse effect on separation in the classical Mortensen's & Pissarides' framework hinges on bilateral wage bargaining. We argue that wage rigidity is a more reasonable assumption for the targeted unemployed in Continental Europe where minimum wage regulations are prevailing. The mechanism whereby separations get stimulated in our model is thus different. It proceeds from the heterogeneity of vacancies and jobs. The model indeed allows for the fact that employers have unequal expectations about the productivity of the job opportunities they face. A type is attached to each job opportunity and reflects this level of expectation. Higher type jobs are on average more productive and are thus more likely to survive beyond the subsidized period and to lead to permanent employment. We show that hiring subsidies decrease the profitability threshold, with lower type vacancies entering the labor market. The well known congestion externalities increase and all vacancies are less easily filled, including high type vacancies. A crowding out effect, by which high type vacancies would be fewer, is also a theoretical possibility.

The model is then calibrated with data from the Southern part of Belgium (Wallonia), a region that constitutes a relatively well circumscribed territory for both labor relations[1] and labor market policy-making and that is characterized by a historically high unemployment prevalence among the young and the unskilled. Simulations reveal that increasing hiring subsidies at the margin from their current level would further decrease unemployment, but this effect would be very limited and net job creations would almost entirely consist of temporary jobs. Our results also indicate that if the subsidy is set too high, then permanent employment would be negatively affected and, to a lower extent, aggregate employment.

[1] At least regarding the unskilled who are less mobile than the skilled whose labor market is larger and includes Brussels and Luxembourg, as discussed below.

The paper is organized as follows: Section 2 discusses how our contribution relates to the existing literature. In Section 3, we develop the analytical model and make comparative statics exercises. Section 4 is devoted to the calibrated model, our case study and the simulations. The implications of our choice of assumptions are discussed in Section 5. Section 6 concludes.

2 Relationship with the literature

Two alternative viewpoints have been adopted to evaluate hiring subsidies in the existing literature: on the one hand, some authors have looked at individual trajectories and have tried to assess whether the hazard rate from unemployment improved as well as longer term outcomes of the beneficiaries. On the other hand, others have adopted a macro viewpoint and have used models to analyze impacts on flows out of and into unemployment and the net impact on employment. With this paper, we obviously want to contribute to the second strand of literature. However, because we distinguish between temporary and permanent employment, we are able to explore two types of individual transitions, from unemployment to employment and then to permanent, stable employment.

If substitution or displacement effects are at play, it might well be that favorable individual outcomes coexist with disappointing outcomes on the aggregate. Matching models allow to analyze labor market flows and to assess net employment effects of hiring subsidies. Mortensen & Pissarides (2001) explore the impact of different kinds of taxes and subsidies, including hiring subsidies, within their classical matching framework. They show that both job creation and job destruction rates increase with hiring subsidies, with a resulting analytical indeterminacy on the net employment effect. The underlying mechanism is as follows. Hiring subsidies stimulate the inflow of vacancies, thereby increasing labor market tightness and job creation. Because exit rates are higher, workers' exit option (as unemployed) is higher and the surplus of a match consequently lower. One could think that the exit option of employers should decrease because a higher tightness makes it more difficult to fill a vacancy. However, the assumption of free entry fixes this value to zero. Because the surplus is lower, jobs are destroyed with a higher probability in the event of a productivity shock. Because in reality job destruction is probably not always jointly decided, this effect can be stated in a more intuitive way: worker with a better outside option command higher wages, profit is lower and the employer is more likely to incur losses if a shock hits. As already mentioned, our effect proceeds differently as we work with exogenous wages. In our model, higher subsidies induce an inflow of vacancies of lower quality, which create jobs whose destruction rate is higher. This result cannot be obtained within the classical Mortensen & Pissarides (1994)'s model where all jobs start at the highest productivity level. This is thus the result of vacancy and job heterogeneity, which we think is a realistic setup and one of our main contributions. Kitao et al. (2011) have recourse to Mortensen & Pissarides (1994)'s model, which they calibrate with US data. They distinguish between the skilled and unskilled segments of the labor market, but do not allow for substitution effects. In their simulations, the destruction effect dominates the creation effect in both skill groups. They highlight that the wage effect, which causes more destructions, is lower on the unskilled labor market whose tightness is initially lower. Jahn & Wagner (2008) also conclude to an increase in unemployment in their model. They point to an increase in the duration of job search if Public Employment Services (PES) are involved, because eligible candidates may crowd out other candidates who search actively, but do not have recourse to the PES. Brown et al. (2011) develop an original model of hirings and separations, from which they draw the conclusion that hiring subsidies targeted at the long term unemployed should be preferred to low wage subsidies to create employment. The reason thereof is that they imply lower deadweight losses and no wage effect. The wage effect is indeed absent for hiring subsidies in their model due to the assumption of a process of collective wage bargaining from which entrants are excluded. The classical effect on destructions is thus absent, like in this paper. However, the rate of job destruction, while endogenous, is equal between entrants and insiders, so that the key firing decision, which takes place at the moment where entrants become insiders, is not explicitly modelled in Brown et al. (2011). Bucher (2010) proposes a model where two unemployment statuses are considered. The distinction is made along unemployment duration. The long term unemployed, who are eligible to the subsidy, suffer from stigmatization and do not have access to high productive jobs. Job destruction is exogenous. Still, hiring subsidies have an ambiguous impact on unemployment. This is due to a substitution effect by which job creations are stimulated among the long term unemployed at the expense of the short term unemployed. The focus on the impact of targeted hiring subsidies on unemployment composition is interesting. It highlights that a higher job turnover is a process by which long term unemployed become short term unemployed, with better prospects. This takes us to the argument that a higher turnover might be socially desirable for equity reasons. A common feature that we share with Bucher (2010)'s model is the result that the creation of low productive jobs is encouraged by the policy. With respect to her paper, our contribution consists in treating the job destruction process as endogenous and in focusing on the composition of employment, not unemployment. With this respect, Cahuc et al. (2016) provide a theoretical analysis of the prevalence of temporary contracts. More precisely, they study employers' choice between temporary and permanent contracts under heterogeneity of expected durations of production opportunities. They show that the prevalence of temporary contracts increases with firing costs, as expected. Because temporary contracts cannot be terminated before their expiration date, employers hit by a productivity shock face losses that depress aggregate production. As compared to their paper, we add incomplete information about the job's characteristics and place heterogeneity on the expected productivity level rather than on the shock arrival rate. Of course, we also look at the effects hiring subsidies, which is out of the scope of their analysis. Notice that many of the notations that we use in what follows are borrowed from Cahuc et al. (2016).

3 The theoretical model

3.1 Assumptions

In this section, we develop a matching model aimed at analyzing the effects of targeted hiring subsidies on employment and employment stability under job heterogeneity. We focus on the targeted segment of the labor market and abstract from substitution or spillover effects that may arise across different segments. Substitutability between targeted workers and others is arguably low if the former are unskilled and/or have a low level of experience, such as the young who just entered the labor market.

The considered segment of the labor market is composed, on the demand side, of a set of infinitely lived, risk-neutral employers. This set can either be discrete or continuous. In other words, as long as it remains compatible with the fact that employers are price-takers (the wage is exogenously set for reasons explained below), a single employer might indifferently hold a unique job, multiple jobs, or no jobs at all. On the supply side, there is a continuum of homogeneous workers of size 1.[2] Workers supply their labor force inelastically, so

[2] As discussed below, the interpretation that workers are homogeneous is the most conservative one but can be partly relaxed

that the size of the active population is exogenously given. There are three labor market statuses $\{U, T, P\}$ as workers can be unemployed U, employed and hold a temporary job T, or hold a permanent position P. We let u, ℓ_t and ℓ_p denote the respective masses of unemployed, temporary and permanent workers or jobs, with $u + \ell_t + \ell_p = 1$. The model is developed in continuous time and transitions between statuses are governed by the following rules: We assume that workers who exit out of unemployment necessary become temporary workers and are automatically entitled to a flow hiring subsidy h, which the employer receives to cover part of the wage w. For simplicity, we assume that the length of the fixed term contract, noted Δ, corresponds to the period during which the subsidy is granted.[3] For any worker-job match formed at time t, transition to permanent employment relies on employers' decision at $t + \Delta$, unless the job has been destroyed for exogenous reasons before. Indeed, both temporary and permanent jobs can be destroyed at any time due to external factors impacting the firm at random. Such shocks follow a Poisson process with arrival rate λ. Permanent jobs are only subject to exogenous destruction since we assume that permanent workers cannot be fired. It should become clear however that, in this model, the employer has no reason to fire a permanent worker he/she has decided to hire, unless an exogenous shock hits the firm, in which case the job is automatically destroyed anyway. Notice that the model does not allow layoffs to take place during the fixed-term contract either, but only once ended (at $t + \Delta$). There are different reasons why the contract end might be seen as a focal point in time for layoffs: In addition to employment protection legislation, this point in time indeed coincides in the model with the exhaustion of the subsidy and with the moment at which the employer can gauge the true productivity of the match, as explained below, and then decide whether it is worth maintaining in the absence of subsidies. We assume that layoffs can take place at no cost, but only at this point in time.[4] As already mentioned, we want to introduce job heterogeneity in the model. To do so, we assume that employers are faced with a continuum of prospective jobs, whose productivity is initially unknown. At any point in time, employers are allowed to draw a signal or job's type, $\varepsilon \sim G(\varepsilon)$ against a lump sum cost κ. This signal gives an indication about their job(s)' future productivity $y \in \mathbb{R}_+$ if opened and filled. This true productivity y will only be observed at the end of the temporary contract, provided the job is filled and still exists at $t + \Delta$. Based on the signal, employers then decide whether to open a vacancy. The informativeness of the signal is the following: Given two signals ε_1 and ε_2, there is a relationship of first order stochastic dominance between $F(y \mid \varepsilon_1)$ and $F(y \mid \varepsilon_2)$, with the former dominating the latter if and only if ε_1 is higher than ε_2:

$$F(y \mid \varepsilon_1) \leq F(y \mid \varepsilon_2), \forall y \in \mathbb{R}_+ \iff \varepsilon_1 \geq \varepsilon_2. \tag{1}$$

Loosely speaking, high type jobs (high values of the signal ε) give better prospects and lead, on average, to higher productivity levels.

Different interpretations are compatible with this informational structure: (1) Workers are homogeneous and y reflects the productivity of the job or task itself. This first interpretation is straightforward. In the model, the most promising jobs, namely jobs whose expected productivity is the highest, will be opened

without affecting the main results of the paper.

[3] Alternatively, one can simply interpret h as a lump sum subsidy received for new hires. This alternative interpretation is equivalent as long as the entrant is not fired in the course of the fixed term contract, which we assume for reasons explained below.

[4] Notice that employment protection legislation might also serve as a justification for the assumption that newly hired workers necessarily start as entrants. Indeed, open-ended contracts may not allow employers to fire employees easily if the productivity of the match is deemed too low.

first. This easily reflects the natural tendency of marginal productivity to be decreasing at the firm and/or at the economy level. (2) Workers are horizontally heterogeneous as well as jobs and the productivity level is match-specific. If the worker's type is private information, the employer does not know a priori whether the worker's skills are adapted to the job's requirements. In this second case, ε can be seen as an indication of the ease or likelihood with which the employer will end up with an appropriate worker. (3) Workers are vertically heterogeneous. In this last case, the productivity of the match y is increasing in both the worker's type and the employer's type ε. An important caveat is that, in this case, low types will be fired first, so that the pool of unemployed will deteriorate over time. Because we also have an exogenous destruction of permanent jobs, some high types will become unemployed as well. While we do not expect this complex process to affect our results qualitatively, we do not explicitly model it in this paper. This last interpretation must therefore be taken with caution.

Let us now summarize the sequence of actions and the timing of information revelation from the viewpoint of an employer:

1. At any point in time, employers can decide to draw a job's type $\varepsilon \sim G(\varepsilon)$ and to incur the associated lump sum cost κ.

2. If they do so, they then get to decide whether to open a vacancy, in which case,

3. the matching process takes place.

4. If a match is formed at time t, then the job is filled by a temporary worker and produces a flow of y units of the numeraire good. Also, the employer receives a flow hiring subsidy h and remunerates the employee at the prevailing wage w. Temporary jobs are exposed to destruction shocks arriving at a rate λ.

5. If the job still exists at $t+\Delta$, then the employer observes his/her job's productivity y and get to decide whether to destroy the job and fire the employee or to turn him/her into a permanent worker.

6. Permanent jobs produce y and are exposed to destruction shocks arriving at a rate λ. Permanent workers are remunerated at the prevailing wage w.

A few additional comments need to be raised.

First, the matching process, as well as its related notations, are standard: Let v denote the vacancy rate and $\theta = v/u$ the degree of tightness on this segment of the labor market. The matching function $M(u,v)$ gives the rate of match formation and is assumed homogeneous of degree 1, with $\partial M/\partial u > 0$ and $\partial M/\partial v > 0$. As a result, vacant jobs are filled at a rate noted $m(\theta) \equiv M/v$, which is decreasing in θ: $m'(\theta) < 0$. Similarly, unemployed workers are hired at a rate $M/u = \theta M/v = \theta m(\theta)$ and is increasing in θ: $(\theta m(\theta))' > 0$.

Second, as already mentioned, we take w as exogenous, thereby considerably simplifying the analysis by ignoring wage effects. It can indeed be argued that wage effects are expected to be negligible in the case of targeted hiring subsidies, for two reasons. On the one hand, because they are granted for a limited period of time, hiring subsidies represent lower amounts as compared to wage subsidies. Their impact on the equilibrium wage should be accordingly lower. On the other hand, our analysis focuses on subsidies that are targeted at vulnerable workers for whom the minimum wage is likely binding. In addition, targeted workers are outsiders, namely initially unemployed and then temporary workers, with low weight in the wage bargaining process.

The third comment pertains to the distinction between temporary and permanent workers. The intuition that is the easiest to grasp is that temporary workers (in the interval $[t, t+\Delta]$ after match formation) work under fixed-term contracts and the others under open-ended contracts. However, this should not be taken literally. The distinction that the model makes between both statuses is based on (1) the information structure and (2) the job destruction process. Regarding the former, we posit that the productivity of a temporary job is unknown. The contract duration (Δ) corresponds to the time needed to fill this information gap. Regarding the latter, the model allows endogenous destruction of temporary jobs at expiration date, while permanent job destruction is purely exogenous. The reason thereof is that we want to capture the incentives faced by employers to destroy employment once the subsidy expires.

3.2 Equilibrium

We solve the model in steady state. To this end, we naturally equalize the flows in and out of each of the three statuses: unemployed U, temporary T and permanent worker P, so as to obtain constant stocks. The distinction between two employment statuses $\{T, P\}$ gives us a modified Beveridge curve, as shown below. The equilibrium of the model is then given by the Beveridge curve and the equilibrium decisions of employers in terms of vacancy supply and temporary job destruction. Let us first analyze the employers' decisions.

3.2.1 Vacancy supply and temporary job destruction

Solving for employers' optimal choices backward, we start by exploring the firing decision. To this end, let us adopt the point of view of an employer holding a temporary job, formed at time t and approaching its expiration date $t+\Delta$, and contemplating whether to offer the worker a permanent position. Considering that the true productivity y is revealed at that date and making use of a Bellman equation, we can write the flow equivalent of the asset value of a permanent job as

$$r\Pi_p\left(y\right) = y - w - \lambda\Pi_p\left(y\right), \tag{2}$$

where r is the discount factor. Notice that the outside option of the employer is equal to zero in equilibrium by free entry (see below). In the absence of firing costs, it is straightforward to see that the employer fires the workers if and only if his/her productivity falls short of the prevailing wage: $\Pi_p\left(y\right) < 0 \iff y < w$. Given that the true productivity is initially unknown, the asset value of a temporary job at the date of match formation is given by $E_{y|\varepsilon}\Pi_t\left(y\right)$, with

$$\Pi_t\left(y\right) = \int_0^{\Delta}\left(y - w + h\right)e^{-(r+\lambda)s}ds + e^{-(r+\lambda)\Delta}\max\left\{\Pi_p\left(y\right), 0\right\}, \tag{3}$$

where the subscript t stands for temporary. The first term of this expression measures the discounted expected profit over the period of the fixed-term contract. The flow profit is constant and takes account of the hiring subsidy: $y - w + h$. With probability $e^{-\lambda\Delta}$, the temporary job will still exist at its expiration date $t+\Delta$, in which case the profit will be equal to the asset value of a permanent job $\Pi_p\left(y\right)$, if positive. Isolating $\Pi_p\left(y\right)$ in equation (2), substituting it into (3), and taking the expectation over the productivity level conditional on the job's type, leads to

$$E_{y|\varepsilon}\Pi_t\left(y\right) = \frac{1}{r+\lambda}\left[\left(1-\delta\right)\int_0^{+\infty}\left(y - w + h\right)dF\left(y \mid \varepsilon\right) + \delta\int_w^{+\infty}\left(y - w\right)dF\left(y \mid \varepsilon\right)\right] \tag{4}$$

where $(1-\delta)$ and δ are the relative weights respectively attributed to the temporary and permanent job in the profit function. δ is defined as

$$\delta \equiv e^{-(r+\lambda)\Delta}.$$

Naturally, we see that the relative weight on the temporary (permanent) contract is increasing (respectively, decreasing) in the length of the temporary contract Δ, in the destruction rate λ and in the discount factor r.

Lemma 1 *The asset value of a temporary job is increasing in the job's type:*

$$\frac{\partial E_{y|\varepsilon}\left[\Pi_t\left(y\right) \mid \varepsilon\right]}{\partial \varepsilon} > 0.$$

Proof. As shown in Appendix 1, this an implication of first order stochastic dominance. ■

Intuitively, this first Lemma simply tells us that, as expected, high type jobs, whose expected productivity is higher, yield higher expected profits.

In a second step, we analyze the vacancy supply behavior. Making use of a Bellman equation, the asset value of a vacancy V satisfies[5]

$$rV\left(\varepsilon\right) = m\left(\theta\right)\left(E_{y|\varepsilon}\Pi_t\left(y\right) - V\left(\varepsilon\right)\right).$$

Isolating V,

$$V\left(\varepsilon\right) = \frac{m\left(\theta\right)}{r + m\left(\theta\right)} E_{y|\varepsilon}\Pi_t\left(y\right).$$

An employer having already invested in a job's type will post a vacancy if and only if it is expected to yield non negative profits:

$$V\left(\varepsilon\right) \geq 0 \iff E_{y|\varepsilon}\Pi_t\left(y\right) \geq 0.$$

Thanks to the monotonicity of $E_{y|\varepsilon}\Pi_t\left(y\right)$ with respect to the type (Lemma 1), we know that there exists a threshold value for the type $\tilde{\varepsilon}$ such that the expected profit of a temporary job and hence the asset value of a vacancy are both equal to zero:

$$V\left(\tilde{\varepsilon}\right) = 0 \iff E_{y|\varepsilon}\left[\Pi_t\left(y\right) \mid \tilde{\varepsilon}\right] = 0. \tag{5}$$

Let us call $\tilde{\varepsilon}$, the vacancy threshold. The most promising jobs, whose type is above the vacancy threshold, are opened. Making use of equation (4), we can also note that

$$E\left[y \mid \tilde{\varepsilon}\right] = w - \left(\delta \int_0^w \left(w - y\right) dF\left(y \mid \tilde{\varepsilon}\right) + \left(1 - \delta\right) h\right) < w.$$

This inequality states that, at the indifference threshold, the expected productivity is actually lower than the wage. This implies that, among opened vacancies, some will yield negative profits, on average. This is due to the option that the employer has to put an end to the temporary contract once expired. The term $\delta \int_0^w \left(w - y\right) dF\left(y \mid \varepsilon\right)$ is indeed positive and corresponds to the loss that the employer can avoid by firing the worker in cases where y turns out to be lower than w. This term is increasing in δ, namely the relative weight attributed to the permanent job in the expression of the expected profit (4), and hence decreasing in the discount factor, the exogenous destruction rate and the length of the temporary contract. This inequality $E\left[y \mid \tilde{\varepsilon}\right] < w$ tells us that the loss potentially incurred on the temporary contract is more than offset by the profits that could yield high productivity levels over an expected longer time horizon. We see that this effect is reinforced by the presence of the hiring subsidy, which could absorb part of this potential loss.

[5] We indeed make the simplifying assumption that the flow cost of a vacancy is equal to zero.

Finally, the free entry condition is as follows:

$$\Pi_J \equiv \int \max \{V(\theta, \varepsilon), 0\} \, dG(\varepsilon) = \kappa. \tag{6}$$

Indeed, employers enter the market (or draw additional types) if doing so is worth the fixed cost. The expected profit of drawing a type Π_J can be rewritten as

$$\Pi_J = \frac{m(\theta)}{r + m(\theta)} \int_{\tilde{\varepsilon}}^{+\infty} E_{y|\varepsilon} [\Pi_t(y) \mid \varepsilon] \, dG(\varepsilon).$$

Because $m'(\theta) < 0$, by assumption, we can easily see that this expression is decreasing in the level of market tightness θ, which ensures that the problem is well-behaved and that condition (6) will be satisfied with equality in equilibrium.

Let us summarize the above reasoning and characterize the vacancy supply in terms of size and composition. Let J denote the number (flow) of job type draws by employers at any time.

Lemma 2 ***Vacancy supply:***

1. *Quality: In equilibrium, opened jobs are such that $\varepsilon \in [\tilde{\varepsilon}, +\infty)$, with $\tilde{\varepsilon}$ such that condition (5) is satisfied with equality.*

2. *Quantity: The equilibrium number of vacancies is given by $v = J(1 - G(\tilde{\varepsilon}))$, where J is such that market tightness $\theta = J(1 - G(\tilde{\varepsilon}))/u$ solves the free entry condition (6), given the vacancy threshold $\tilde{\varepsilon}$.*

Proof. This results from the above paragraph. ■

Because of job heterogeneity, the probability that a temporary worker be fired at the end of the fixed-term contract is job type-specific. Recalling that a worker is fired whenever $y < w$, this probability is simply equal to $F(w \mid \varepsilon)$. Averaging over the jobs that are opened and filled, we find that the destruction rate of temporary jobs, which still exist at expiration date, is given by[6]

$$\bar{F}(\tilde{\varepsilon}) \equiv E_{\varepsilon \geq \tilde{\varepsilon}} [F(w \mid \varepsilon)]. \tag{7}$$

Naturally, high types have a lower destruction probability because their productivity y is more likely to be higher than the wage, by first order stochastic dominance: $\partial F(w \mid \varepsilon)/\partial \varepsilon < 0$. An important implication of this property is that, for given J and w, temporary job destruction $\bar{F}$ is negatively related to the vacancy threshold: $\partial \bar{F}(\tilde{\varepsilon})/\partial \tilde{\varepsilon} < 0$ (see Appendix 2 for a formal proof). As a result, holding the number of type draws J constant, more vacancies are necessarily associated with a higher rate of job destruction. As will be discussed at length below, the extent of the positive impact of hiring subsidies on job destruction is strongly dependent on how they stimulate the vacancy supply. If this is mainly by leading employers to open low type vacancies, this positive impact on job destruction will be stronger.

3.2.2 The Beveridge curve

HERE: add a graph with stocks and flows.

Let us now establish the flow equations, which ensure constant stocks of unemployed u, temporary ℓ_t, and permanent workers ℓ_p.

[6] Indeed, we assume that the probability to fill a vacancy is orthogonal to the job type.

First, the number of unemployed u is constant over time if and only if

$$\lambda \left(\ell_p + \ell_t\right) + \bar{F}\frac{e^{-\lambda\Delta}}{\Delta}\ell_t = \theta m\left(\theta\right) u. \tag{8}$$

The left hand side of (8) measures the flow into unemployment. It is composed of exogenous and endogenous job destruction. The former is given by the first term on the left hand side of condition (8). The latter corresponds to the second term. Notice that $\left(e^{-\lambda\Delta}/\Delta\right)\ell_t$ measures the number of surviving temporary jobs approaching expiration date at any time. Among them, a fraction $\bar{F}$ is destroyed following employers' decision. Naturally, the right hand side is equal to the flow out of unemployment, which is given by the job finding rate $\theta m\left(\theta\right)$ times the number of unemployed u.

Second, in steady state, there is equality between entries into and exits out of temporary employment:

$$\theta m\left(\theta\right) u = \left(\lambda + \frac{e^{-\lambda\Delta}}{\Delta}\right)\ell_t. \tag{9}$$

The flow of entries into temporary employment is equal to the flow out of unemployment, by assumption (the left hand side of (9)). The flow of exits is composed of the flow of exogenous destructions $\lambda\ell_t$ and of the number of surviving contracts coming to an end at each point in time. Indeed, the latter, which are either endogenously destroyed or converted into permanent employment, disappear from the count of temporary employment anyway.

Third and finally, the number of permanent jobs ℓ_p is constant over time if an only if

$$\left(1-\bar{F}\right)\frac{e^{-\lambda\Delta}}{\Delta}\ell_t = \lambda\ell_p. \tag{10}$$

The flow into permanent employment is given by the number of surviving temporary jobs which are converted into permanent employment (the left hand side of (10)). The flow out of it is simply given by the extent of exogenous permanent job destruction (the right hand side of (10)).

Lemma 3 *The Beveridge curve is written as follows:*

$$u = \frac{\sigma}{\theta m\left(\theta\right) + \sigma}, \tag{11}$$

where

$$\sigma \equiv \lambda\frac{\lambda\Delta + e^{-\lambda\Delta}}{\lambda\Delta + \left(1-\bar{F}\right)e^{-\lambda\Delta}}. \tag{12}$$

Proof. This relationship is obtained by combining the flow equations ((8), (9) and (10)). ■

Equation (11) is an extension of the standard Beveridge curve to the case where there are two distinct employment statuses, as well as two sources of job destruction. In the extreme case where the firing rate $\bar{F}$ is equal to zero, it is easy to see that σ boils down to λ, indicating that we are back to the standard case with a single employment status and purely exogenous job destruction. Notice also that the rate of job destruction σ is increasing in the firing rate $\bar{F}$, as expected.

In sum, the labor market equilibrium is fully characterized by a triple $(\tilde{\varepsilon}, \theta, u)$, such that the equilibrium conditions for vacancy supply (vacancy threshold and free entry) and the Beveridge curve are satisfied:

$$\begin{aligned}
\tilde{\varepsilon} &\;:\; E_{y|\varepsilon}\left[\Pi_t\left(y\right) \mid \tilde{\varepsilon}\right] = 0, \\
\theta &\;:\; \frac{m\left(\theta\right)}{r + m\left(\theta\right)}\left(1 - G\left(\tilde{\varepsilon}\right)\right) E_{\varepsilon\geq\tilde{\varepsilon}} E_{y|\varepsilon}\left[\Pi_t\left(y\right) \mid \varepsilon\right] = \kappa, \\
u &\;:\; u = \frac{\sigma}{\theta m\left(\theta\right) + \sigma},
\end{aligned}$$

where σ depends on the firing rate $\bar{F}$, which itself depends on $\tilde{\varepsilon}$ (see equations (12) and (7)). Notice that θ actually determines the equilibrium level of J, which determines the mass of vacancies (see Lemma 2) for any given rate of unemployment u.

3.3 Comparative statics

This sub-section is devoted to a comparative statics exercise. We naturally want to explore the impact of hiring subsidies h on unemployment, employment composition (ℓ_t and ℓ_p) and on the size of the flows between the 3 statuses, which determine the probabilities of the different trajectories that a worker might follow. To this end, we need to calculate how the labor market equilibrium, defined by the triple $(\tilde{\varepsilon}, \theta, u)$, is affected by a change in the hiring subsidy h.

Proposition 1 *A higher level of hiring subsidy*

1. *decreases the vacancy threshold:* $d\tilde{\varepsilon}/dh < 0$*,*

2. *increases the level of market tightness:* $d\theta/dh > 0$*.*

Proof. Provided in Appendix 3. ■

This proposition explores the effect of a marginal increase in the hiring subsidy on the supply of vacancies. We first note that the vacancy threshold gets reduced. Recall that this threshold determines the range of job types that are opened $[\tilde{\varepsilon}, +\infty)$ among jobs whose type has been revealed following the employers' decision to enter the market. The interpretation of this effect is straightforward as it is simply due to the increase in the asset value of temporary jobs. It results that, on average, jobs that are opened are of lower quality, which means that their destruction rate will be higher.

The second point of the proposition establishes that market tightness increases. The interpretation is similar as it results from the increase in the asset value of temporary jobs, which gives employers incentives to draw additional job types. Technically speaking, this effect is given by the impact of hiring subsidies on the free entry condition (6). This impact is direct, through the asset value of a temporary job and indirect, through the change in the vacancy threshold. However, given that the expected profit is equal to zero in the neighborhood of this threshold, the impact of the decrease in $\tilde{\varepsilon}$ on the expected profit of entering the market Π_J is equal to zero.

Proposition 2 ***Effects on job creation and job destruction:***

1. *The rate of job creation increases with the hiring subsidy:* $d\left(\theta m\left(\theta\right)\right)/dh > 0$*.*

2. *The rate of job destruction increases with the hiring subsidy:* $d\sigma/dh > 0$*.*

Proof. The first point is a direct implication of the second point of Proposition 1. Regarding the second point, the derivative of σ with respect to h is given by

$$\frac{d\sigma}{dh} = \frac{\partial\sigma}{\partial\bar{F}}\frac{\partial\bar{F}}{\partial\tilde{\varepsilon}}\frac{d\tilde{\varepsilon}}{dh},$$

where $\partial\sigma/\partial\bar{F} > 0$, by equation (12), $\partial\bar{F}/\partial\tilde{\varepsilon} < 0$ (see Appendix 2) and $d\tilde{\varepsilon}/dh < 0$, by Proposition 1. ■

Corollary 1 *The impact of the hiring subsidy on unemployment has an indeterminate sign.*
Proof. *This can be seen from the Beveridge curve (11) and from Proposition 2.* ■

The effects of the hiring subsidy on the endogenous variables pertaining to the supply of vacancies $(\tilde{\varepsilon}, \theta)$ allow us to determine the impacts on job creation and job destruction. The increase in market tightness naturally leads to a higher rate of job creation. However, the rate of job destruction increases as well, so that the net effect on unemployment is a priori indeterminate. This adverse effect of hiring subsidies emerges owing to presence of both endogenous job destruction and job heterogeneity. As mentioned earlier, hiring subsidies allow lower job types to be profitable, which result in an increase in job destruction at expiration date. The mechanisms that we have highlighted up to this point of the analysis are relatively intuitive. More jobs are created and these additional jobs incur a higher risk of destruction. One might conclude from this that the mass of temporary jobs should inflate and that labor turnover should be higher. Intuitively however, it is not straightforward to see why permanent jobs would be affected by hiring subsidies. In other words, one might think that hiring subsidies create unstable jobs but have no impact on jobs that would have been created in the absence of subsidies. While exploring the impact on job composition, we will see that this statement is wrong.

Let us now turn to the central question of this paper, which pertains to the impact of hiring subsidies on labor market trajectories and employment composition. Different trajectories are analyzed. We essentially aim at assessing whether exists out of unemployment are durable. To this end, let us distinguish between two cases: either the worker reaches permanent employment, or he/she does not, his/her job being destroyed for exogenous reasons during the temporary contract or following the employer's decision at expiration date. We note that endogenous job destruction creates a discontinuity in the survival function, which writes

$$\begin{aligned} P(S > s) &= e^{-\lambda s}, \text{ if } s \in [0, \Delta), \\ &= e^{-\lambda s}\left(1 - \bar{F}\right), \text{ if } s \in [\Delta, +\infty). \end{aligned}$$

where S denotes survival, namely the duration of the employment spell. Therefore, the probability to reach permanent employment evaluated at the date of hiring is given by

$$P(S > \Delta) = e^{-\lambda\Delta}\left(1 - \bar{F}\right).$$

Hence, the probability that the employment spell be at most equal to Δ is

$$P(S \leq \Delta) = 1 - P(S > \Delta) = 1 - e^{-\lambda\Delta}\left(1 - \bar{F}\right).$$

The following proposition analyzes the impact of hiring subsidies on labor market trajectories.

Proposition 3 ***Effect on trajectories:*** *If the hiring subsidy increases at the margin, then*

1. *the probability to exit out of unemployment* $\pi_{UT} \equiv \theta m(\theta)$ *increases:* $d\pi_{UT}/dh > 0$*;*
2. *the probability to exit out of unemployment temporarily (having an employment spell of at most* Δ*)* $\pi_{UTU} \equiv \theta m(\theta) P(S \leq \Delta)$ *increases:* $d\pi_{UTU}/dh > 0$*;*
3. *the sign of the impact on the probability to exit out of unemployment and to reach permanent employment* $\pi_{UTP} \equiv \theta m(\theta) P(S > \Delta)$ *is indeterminate.*

Proof. These results are direct implications of Proposition 1 and the fact that $\partial\bar{F}/\partial\tilde{\varepsilon} < 0$ (see Appendix 2). ■

The main implication of these results is that labor turnover increases with hiring subsidies. Besides, it is not clear whether the flow from unemployment to permanent employment gets higher. Impacts on employment composition are analyzed below.

Proposition 4 ***Effect on employment composition:*** *The proportion of temporary jobs over total employment increases with the hiring subsidy:* $d\left(\ell_t/\left(\ell_t+\ell_p\right)\right)/dh > 0$.

Proof. Provided in Appendix 4. ■

It appears clearly that the proportion of temporary employment increases.

What remains unclear is whether permanent employment is positively or negatively impacted by the hiring subsidy. In order to have more insights into this effect, let us analyze it in more details. We show in Appendix 4 that the mass of permanent jobs can be written as

$$\ell_p = \rho \int_{\tilde{\varepsilon}}^{+\infty} m\left(\theta\right) J \left(1 - F\left(w \mid \varepsilon\right)\right) dG\left(\varepsilon\right), \tag{13}$$

where

$$\rho = \frac{1}{\lambda} \frac{e^{-\lambda\Delta}}{\lambda\Delta + e^{-\lambda\Delta}}.$$

The parameter ρ captures what relates, in ℓ_p, to the process of exogenous job destruction. This process left aside, we can see that the mass of permanent jobs is given by the integral, over all job types that lead to a vacancy $\varepsilon \in [\tilde{\varepsilon}, +\infty)$, of the number of jobs that are opened $Jg\left(\varepsilon\right)$, filled at a rate $m\left(\theta\right)$ and converted into permanent employment, which occurs with a probability of $1 - F\left(w \mid \varepsilon\right)$. We can decompose the marginal impact of hiring subsidies on this expression (13) in the following way:

$$\frac{d\ell_p}{dh} = \rho \left[\begin{array}{c} m'\left(\theta\right) \frac{d\theta}{dh} J \int_{\tilde{\varepsilon}}^{+\infty} \left(1 - F\left(w \mid \varepsilon\right)\right) dG\left(\varepsilon\right) \\ -m\left(\theta\right) J \left(1 - F\left(w \mid \tilde{\varepsilon}\right)\right) g\left(\tilde{\varepsilon}\right) \frac{d\tilde{\varepsilon}}{dh} \\ +m\left(\theta\right) \frac{dJ}{dh} \int_{\tilde{\varepsilon}}^{+\infty} \left(1 - F\left(w \mid \varepsilon\right)\right) dG\left(\varepsilon\right) \end{array} \right]. \tag{14}$$

Three effects are at play. Two of them relate to the re-composition of the supply of vacancies and one to their filling rate. The latter effect is captured by the first term on the right hand side of (14). It states that market tightness increases $d\theta/dh > 0$ and that opened vacancies are less easy to fill $m'\left(\theta\right) < 0$. This first effect is a congestion externality effect, by which the inflow of new vacancies reduces the filling rate of all vacancies, thereby reducing the number of jobs that will turn into permanent positions. The second term on the right hand side of (14) tells us that lower type jobs are opened $d\tilde{\varepsilon}/dh < 0$. However, the impact on the number of job type draws J, captured by the third term, is indeterminate. Indeed, recall from Lemma 2 that J adjusts so as to achieve a certain level of market tightness, itself compatible with the free entry condition (6). Two cases can be encountered as J can be positively or negatively affected. The former case is the most intuitive: in this case, hiring subsidies stimulate job openings, not only by adding vacancies of lower type, but also by increasing the number of high type vacancies. Because of the congestion externality, this does not guarantee, however, that more high type jobs will be filled. Keeping in mind that high type jobs are more likely to lead to permanent employment, the impact on ℓ_p remains ambiguous. The latter situation is even less favorable to the creation of permanent jobs. Indeed, it is theoretically possible that the increase in market tightness results from an important decrease in the vacancy threshold $\tilde{\varepsilon}$, namely an massive inflow of low type vacancies. In such a case, one cannot exclude that $dJ/dh < 0$, entailing a reduction in high type vacancies. In other words, more vacancies are indeed opened, but low type vacancies crowd out high type vacancies. Because low type jobs are less likely to lead to permanent employment, the mass of permanent jobs could be negatively affected by hiring subsidies. This crowding out mechanism and/or the congestion externality are the reasons why stimulating the number of vacancies might be detrimental to the number of permanent jobs. The numerical simulations presented in the next Section will allow us to provide insights on the likely impact of hiring subsidies on the number of permanent jobs.

4 The calibrated model

4.1 Specifying the model

In order to perform numerical simulations of the model displayed in the preceding Section, we need to further specify (1) the information structure and (2) the matching technology.

Regarding the information structure, we assume that productivity is (unconditionally) distributed over the closed interval $[0, \bar{y}]$; that the distribution of productivity conditional on the signal is uniform: $y \mid \varepsilon \sim U[y_L(\varepsilon), y_H(\varepsilon)]$; and that, for simplicity, the signal itself is also uniformly distributed: $\varepsilon \sim U[\varepsilon_L, \varepsilon_H]$. Having both the infimum and the supremum of $y \mid \varepsilon$ monotonically increasing in the signal ε ($\partial y_L / \partial \varepsilon > 0$ and $\partial y_H / \partial \varepsilon > 0$) is sufficient to guarantee first order stochastic dominance between $y \mid \varepsilon_2$ and $y \mid \varepsilon_2$, for all $\varepsilon_2 > \varepsilon_1$ (see (1)). As intuition suggests, for a given interval $[0, \bar{y}]$ of the unconditional distribution, the respective sizes of the supports of $y \mid \varepsilon$ and ε are related to the extent to which the employer is uncertain about his/her job's true productivity y. The smaller the support of the conditional distribution $y \mid \varepsilon$ and the larger the support of the signal ε, the smaller this uncertainty. We capture this relationship with a parameter $\alpha \in [0, 1]$, which influences both supports in the following way:

$$\varepsilon_L = (1-\alpha)\frac{\bar{y}}{2}, \tag{15}$$

$$\varepsilon_H = (1+\alpha)\frac{\bar{y}}{2}, \tag{16}$$

$$y_L(\varepsilon) = \varepsilon - (1-\alpha)\frac{\bar{y}}{2}, \tag{17}$$

$$y_H(\varepsilon) = \varepsilon + (1-\alpha)\frac{\bar{y}}{2}. \tag{18}$$

With this specification, the sizes of both supports are

$$\varepsilon_H - \varepsilon_L = \alpha \bar{y}, \tag{19}$$

$$y_H - y_L = (1-\alpha)\bar{y}. \tag{20}$$

This parameter α is positively related to the informativeness of the signal. This can be easily understood by examining the two extreme cases. On the one hand, if $\alpha = 0$, then the distribution of the signal is degenerated with $\varepsilon \sim U[\bar{y}/2, \bar{y}/2]$ and $y \mid \varepsilon \sim U[0, \bar{y}]$. In this first case, the signal has no informative value. On the other hand, in the opposite case where $\alpha = 1$, we have that $\varepsilon \sim U[0, \bar{y}]$ and $y \mid \varepsilon \sim U[\varepsilon, \varepsilon]$. In this second case, the true productivity is known to the employer once ε is drawn. More generally, α therefore measures the fraction of uncertainty that is resolved once the signal is revealed to the employer.[7] It can be useful to note that our specification implies that the conditional expectation of productivity is simply equal to the signal: $E[y \mid \varepsilon] = \varepsilon$.

[7] Notice also that this specification also ensures that the lowest and the highest possible values of y are precisely equal to zero

Let us now turn to the matching technology. As is standard in the literature, we have recourse to a Cobb-Douglas matching function:

$$M(u,v) = Au^{\eta}v^{1-\eta},$$

with A the total factor productivity of matching and η the elasticity of the number of matches to the stock of unemployed. This choice implies that the probability to fill a vacancy and the probability to exit out of unemployment are respectively given by

$$\begin{aligned} m(\theta) &= A\theta^{-\eta}, \\ \theta m(\theta) &= A\theta^{1-\eta}. \end{aligned}$$

In addition to the use of the above functional forms, the calibrated model differs from the theoretical model in that it is developed in discrete time, so as to correspond to the structure of the data. Details can be found in Appendix 5.

4.2 The case of Wallonia

In the present Section, we calibrate the model so as to match the labor market flows observed in Wallonia, the southern, French-speaking, part of Belgium. Wallonia offers a good case study for analyzing the effects of targeted hiring subsidies for the following reasons.

First, because Belgian regions are small and relatively densely populated, daily commuting is generally feasible. In addition, with the noteworthy exception of Brussels, which attracts skilled workers from both neighboring regions, Belgian regions, namely Flanders and Wallonia, have relatively separated labor markets, owing to linguistic barriers. There are also structural differences between regions in terms of economic performance and characteristics of the labor market (Bodart et al. (2018)). As Bodart et al. (2018) conclude, national boundaries are inappropriate to serve as a unit of analysis of the Belgian labor market. Moreover, cross-border commuting, while important between Wallonia and Luxembourg is limited to high-skilled workers, who are out of the scope of the policy under study. For these reasons, Belgian regions offer geographically integrated and circumscribed labor markets, at least as far as unskilled labor is concerned.

Second, some important labor market policies, including hiring subsidies and more generally the so-called *target group policy*, are precisely conducted at the regional level since the 6th institutional reform which has been implemented between 2012 and 2014. Belgian regions are therefore appropriate units of analysis because their labor market and territory for policy-making coincide at the same geographical scale.

The third reason, which applies more specifically to Wallonia, is that unemployment is largely prevalent among vulnerable workers, particularly the young and the unskilled, who are the main targets of hiring subsidies. Indeed, the youth unemployment rate amounted to 29% in 2017 according to Eurostat LFS data (among the less than 25 active workers), which follow the ILO definition. This high regional rate compares to a corresponding rate of 19.3% at the national level, the latter being almost equal to the EU average (without

and $\bar{y}$, respectively:

$$\begin{aligned} y_L(\varepsilon_L) &= \varepsilon_L - (1-\sigma)\frac{\bar{y}}{2} = 0, \\ y_H(\varepsilon_H) &= \varepsilon_H + (1-\sigma)\frac{\bar{y}}{2} = \bar{y}. \end{aligned}$$

Indeed, due to the strict monotonicity of y_L and y_H with respect to ε, the infimum of the unconditional distribution of y can only be achieved if $\varepsilon = \varepsilon_L$, while the supremum can only be achieved if $\varepsilon = \varepsilon_H$.

the UK).[8] If one looks at the unskilled, we see that 20.1% of them are unemployment in 2016 as compared to 16.1% for Belgium, which is again very close to the EU average. Again according to 2017 LFS data, the prevalence of fixed term contracts and other temporary statuses is relatively high in Wallonia with 11.8% against 10.4% for Belgium as a whole. The difference is more marked among the young with 53.6% at the regional level against 47.3% at the country level. This gives a first raw indication that employment stability is particularly limited among the young. Our theory suggests that hiring subsidies may decrease unemployment at the expense of employment stability. These figures therefore tend to indicate that this tradeoff might be especially tight in the case of Wallonia.

If we limit ourselves to the two main schemes available to employers in 2018, we see that hiring subsidies in Wallonia are targeted at two categories of workers, namely the young and the long term unemployed.[9] The first scheme is called *Impulsion moins de 25 ans* and relates to the former category. As the noun of the measure indicates, eligible workers are indeed aged under 25. But, in addition to age, education and duration in unemployment also belong to the set of eligibility criteria. Indeed, eligible individuals should hold at most a high school degree. The subsidy is immediately available to the unskilled young (strictly less than high school) and available after 6 months in unemployment for the others (high school degree). The policy consists of a wage subsidy granted during 3 years in total, with a decreasing time profile. The subsidy amounts to 500 euros per month during the first two years. It is then respectively reduced to 250 and 125 euros per months for the last two periods of 6 months. The second scheme is called *Impulsion 12 mois +*. It is available to all workers who have been unemployed for at least 12 months. The full wage subsidy amounts to 500 euros per month and is granted during one year. The same decreasing pattern as the one explained for the first measure applies during the second year.

Our calibration strategy relies on data from the Crossroads Bank for Social Security (CBSS), which centralizes administrative data on workers and allowed us to calculate a series of transition probabilities to be matched by the different flows and/or probabilities appearing in the model: the destruction rate of permanent employment, the exit rate out of unemployment and the survival rate until permanent employment (see Appendix 6 for more details). In the calibration, we focus on the young (less than 25). It is important to note that the concept of administrative unemployment differs from the ILO's definition of unemployment. Indeed, administrative unemployment records all workers who are registered as job seeker at the public employment service (PES). However, among them, some are actually either unavailable for a job and/or not actively looking for a job. Also, some job seekers who fulfil the ILO's criteria may be, for different reasons, not registered at the PES. This is the first reason why the unemployment rate that appears in the baseline calibration is different (lower) than the figure mentioned above, which is based on survey data. The second reason is that we derive the baseline unemployment from the Beveridge curve, based on transition rates. As a result, we make the assumption that the economy is at a steady state, which might not be true in reality. In order to mitigate this difficulty, we take averages over several years of observations as detailed in Appendix 6.

Table 1 presents the transition probabilities as estimated from the CBSS as well as parameter values that

[8] It is standard to use the ILO definition for international comparisons. For reasons briefly explained below, this rate differs significantly from the unemployment rate calculated on administrative data, which we use in the simulation exercises.

[9] It should be noted that hiring subsidies have been reformed in July 2017 in Wallonia. However, the former scheme, while more complicated, was qualitatively similar. As a result, even though the data that we use for simulation are older than the reform, we do not expect this discrepancy to influence our results. The current scheme is easier to use as a description of the local policy.

are used in the baseline calibration.

HERE: insert table 1.

4.3 Results

Figures 1 to 3 plot the transition probabilities and the prevalence of the three labor market statuses $\{U, T, P\}$ against the level of hiring subsidy, measured as a percentage of the exogenous wage. The hiring subsidy currently represents about 13% of the wage (see Appendix 6). As explained at length in Appendix 6, the model is calibrated from this point and parameters are fixed so that the model replicates the labor market transition rates found in the data. These relationships are derived for three different values of α, which measures the degree of informativeness of the signal. Note that if α is higher, then the support of the conditional distribution of productivity is reduced as can be seen from equation (20), which means that the probability to observe a productivity level that significantly deviates from the signal is lower. The consequences of this effect are discussed below.

Results of the calibrated model - hopefully - confirm what we already knew from comparative statics, namely that both the rate of job creation $\theta m(\theta)$ and the separation rate $\bar{F}$ are increasing functions of the hiring subsidy. What are then the new insights provided by simulations?

First, simulations give us insights into the shape of these functions. The rate of job creation appears to be everywhere convex in the subsidy, while the separation rate is first convex and might become concave.

The impact of the hiring subsidy on unemployment of course depends on these two rates, recalling that a process of exogenous job destruction is also present. We observe in the 3 variants (Figures 1 to 3) that the impact on unemployment is limited. This is the second contribution of our simulations. In the neighborhood of the current value of 13%, we can see that the job creation effect dominates the adverse effect on separations, so that unemployment would decrease if the subsidy would be set higher. The impact on unemployment is however non-monotonic in the 3 variants. At some point, unemployment reaches a minimum and then starts rising again. Looking at the baseline calibration where α is set at 0.6, it can be seen that this minimum corresponds to a hiring subsidy of about 30% of the wage. In the two other variants where α is set at 0.5 and 0.7, the minimum is reached at 50% and 20%, respectively. When comparing the baseline level of unemployment to its minimum, we observe that the - maximal - reduction is very modest as it varies between 0.06 and 0.42 percentage points, depending on the variant. More importantly, it should be noted that this minimum is not socially optimal. Indeed, at this point, the marginal benefit in terms of unemployment reduction is equal to zero, while the marginal cost is strictly positive as all new hires among the target population are entitled to the - marginally higher - subsidy. In the other direction, one might want to have an estimate of the employment effect of the current policy. To this end, we consider our estimate of the counterfactual situation where subsidies are absent. Depending on the value of α, we find that the reduction in unemployment allowed by the current policy is estimated to lie between 0,25 and 0,47 percentage points. The strongest effect is found for α equal to 0.7.

The third contribution of the simulations is to highlight the effect on employment composition. As expected, temporary employment increases with the hiring subsidy across all variants of the calibrated model. The theory was inconclusive about the impact on permanent employment. Simulations reveal that it is barely affected by subsidies set at reasonable levels. In particular, reduction in permanent employment is insignificant in the neighborhood of the current level of 13%. The decrease becomes sharper only at higher levels, essentially beyond the point where unemployment is minimized. So, as expected, higher levels of the subsidy induce a change in employment composition and the relative prevalence of permanent employment declines. However, the increase in the ratio of temporary employment over total employment that can be attributed to the policy is very limited as it is estimated to be lower than 1% in all cases. Again, this estimation can be done by comparing the prevailing rate with the counterfactual where the subsidy is set to zero.

Let us now discuss the role of the parameter α. In light of figures 1 to 3, we can conclude that a higher α exacerbates the effect of the hiring subsidy on transition rates. In particular, the separation rate reaches the highest levels under the assumption that α is equal to 0.7. Recall that the jobs' true productivity is initially unknown to employers. The fraction of uncertainty that is resolved at the time of job opening is precisely given by α. Put differently, α is inversely related to uncertainty about the productivity level among temporary jobs. The higher this uncertainty, the higher the likelihood that the productivity level be higher (lower) than the wage "by accident" among low (high) types. In other words, low (high) type jobs may turn out to be (un)profitable with a higher probability. In the configuration where the remaining uncertainty is important (α is low), all sorts of jobs may lead to permanent employment. When α is high, the converse is true. In the latter case, the crowding out effect by which low types vacancies are stimulated by the hiring subsidies and reduce the filling rate of high type vacancies is more problematic. This is confirmed by the behavior of the separation rate, which goes higher under the assumption that α is equal to 0.7. We see that this is also the case where the decline in permanent jobs is the most important. It is however difficult to approach α empirically. But we can conclude that an environment that is characterized by a low level of uncertainty is more prone to the replacement of permanent jobs by temporary jobs. There is however an alternative interpretation that we need to consider. Indeed, we can see from equation (19) that α is also a measure of job heterogeneity. In the extreme case where α is equal to zero, all jobs are initially alike and hiring subsidies cannot foster low quality vacancies. Job heterogeneity is therefore crucial to the effects studied in this paper.

5 Discussion

This section is devoted to a discussion of two important points, which pertain to the theory.

First, we need indeed to acknowledge that the displacement effect, which consists of the positive impact of hiring subsidies on separations and on which the model focuses, may not be the one that spontaneously comes to mind. Indeed, one might think that employers who fire the employee at the end of the subsidized period do so in order to benefit from the hiring subsidy once again with a new employee. This is not true in this model. Indeed, owing to the assumption of free entry, an employer who would destroy his/her job and re-enter the market makes zero profit, even in the presence of subsidies and because of the presence of competitors who enter until expected profits are exhausted. This is why the firing decision only hinges on profits that could be make in the future with the current match. Ultimately, this is due to the assumption that the type is attached to the job, not to the employer, and disappears in the event of separation, like in Cahuc et al. (2016).

The situation would be different if the employer were to keep the type after separation. Given that creating a job, that is drawing a type, is costly, incumbent employers would have a rent. This rent would increase the productivity threshold, equal to the wage and hence constant across employers in the current model, under which separation takes place. The separation rate would then be higher across all types. However, and more importantly, this effect would be heterogeneous. Indeed, given that the expected profit of a match is increasing in the job's type, this rent would be positively related to the type. It follows that high types would have a stronger incentive to fire temporary workers at expiration date. More precisely, high types would have a higher productivity threshold and thus require more from their temporary employees to keep them, but at the same time a higher probability to be above a given threshold. As a consequence, the pattern of separation, namely relationship between the type and the separation rate would not be necessarily monotonic. We believe that this alternative set of assumptions would unnecessarily complicate the analysis, mainly because we think that having a higher separation rate among low types is empirically plausible. Moreover, notice that the model also abstracts from learning effects. With the latter, the level of productivity would increase over time. The question is how they interact with the job's type. One might reasonably assume that high type jobs require and/or imply more learning. In other words, learning and the job's type are complements. If this is true, then newly hired workers are less productive, which weakens the firing incentive faced by employers holding high type jobs. Learning effects may then well induce less separations among high types, which is portrayed in a simpler way in the current model.

Second, as amply discussed in Section 2, the classical bilateral wage bargaining setting would have impacted our results. More precisely, it would have reinforced the adverse effect of hiring subsidies on separations. Indeed, equilibrium wages would have increased as a result of a higher exit option for eligible workers. Because of its lack of realism for disadvantaged workers in the European context, we have avoided this unnecessary complication.

6 Conclusion

In this paper, we have studied the implications of job heterogeneity for the displacement effect of hiring subsidies in a model with temporary and permanent employment. We have shown that, under this realistic assumption, hiring subsidies stimulate separations by flooding the market with lower quality vacancies, which are less likely to give rise to permanent employment. It results that, while the rate of job creation increases, the impact of hiring subsidies on unemployment is indeterminate. In terms of individual trajectories, this effect translates into a higher exit rate out of unemployment, but also a higher probability of erratic trajectories where workers go back and forth between unemployment and temporary employment. The effect on the probability to exit out of unemployment permanently is indeterminate. Finally, regarding employment composition, our theory predicts that the proportion of temporary employment should be higher when hirings are subsidized, as expected. The impact on the mass of permanent jobs cannot be signed.

We point out that there might be a quantity - quality tradeoff that policy makers should be aware of when deciding to stimulate the supply of job vacancies. Indeed, our contribution highlights that, if lower type vacancies enter the market, then higher type vacancies are negatively affected by congestion externalities. In extreme cases, lower type vacancies may even crowd out higher type vacancies.

It is however challenging to have an estimate of the productivity of marginal jobs, those whose creation is attributable to the policy. We therefore calibrate our model with data from Wallonia. Simulations reveal a favorable, but limited, impact of the prevailing policy on unemployment with almost no negative consequence for permanent employment. However, they suggest that net job creation almost entirely consists of temporary jobs. Finally, we do not recommend to increase the subsidy level too much as marginal gains would be low and at some point may be negative.

Further research would be useful to assess welfare effects of the policy. Indeed, the crowding out effect has also negative impacts on aggregate production given that lower productive jobs are created, possibly at the expense of higher productive jobs. This effect would deserve specific attention. Besides, the calibrated model is unable to replicate the exceptionally high ratio of temporary employment observed in the data, an issue that should be further investigated.

Finally, despite the moderately optimistic message conveyed by this paper, it is fundamental to acknowledge that hiring subsidies may be desirable as a social policy. Indeed, considering that a higher job turnover allows more people to benefit from a work experience, individual jobs' prospects might eventually improve beyond what is captured in the model, mainly among the most vulnerable workers. In this view, it might still be helpful to equalize job opportunities even though employment effects are disappointing.

References

Blundell, R., Dias, M. C., Meghir, C., & Reenen, J. V. (2004). Evaluating the employment impact of a mandatory job search program. *Journal of the European Economic Association*, 2(4), 569–606.

Bodart, V., Dejemeppe, M., & VanderLinden, B. (2018). The labor market in belgium, 2000-2016. *IZA World of Labor*, (428).

Brown, A. J., Merkl, C., & Snower, D. J. (2011). Comparing the effectiveness of employment subsidies. *Labour Economics*, 18(2), 168–179.

Bucher, A. (2010). Impacts of hiring subsidies targeted at the long-term unemployed on the low-skilled labor market: The french experience. *Economic Modelling*, 27(2), 553–565.

Cahuc, P., Carcillo, S., & Le Barbanchon, T. (2014). *Do Hiring Credits Work in Recessions? Evidence from France*. IZA Discussion Papers 8330, Institute for the Study of Labor (IZA).

Cahuc, P., Charlot, O., & Malherbet, F. (2016). Explaining the spread of temporary jobs and its impact on labor turnover. *International Economic Review*, 57, 533–572.

Cockx, B., Gobel, C., & VanderLinden, B. (2004). *Politiques d'activation pour des jeunes chomeurs de longue duree sans experience de travail*. Technical report, Politique scientifique federale, Bruxelles & Academia Press, Gand.

Dahlberg, M. & Forslund, A. (2005). Direct displacement effects of labour market programmes. *Scandinavian Journal of Economics*, 107(3), 475–494.

Jahn, E. J. & Wagner, T. (2008). *Do Targeted Hiring Subsidies and Profiling Techniques Reduce Unemployment?* IZA Discussion Papers 3768, Institute for the Study of Labor (IZA).

Kangasharju, A. (2007). Do wage subsidies increase employment in subsidized firms? *Economica*, 74(293), 51–67.

Kitao, S., Sahin, A., & Song, J. (2011). Hiring subsidies, job creation and job destruction. *Economics Letters*, 113(3), 248–251.

Mortensen, D. T. & Pissarides, C. (2001). *Taxes, Subsidies and Equilibrium Labour Market Outcomes.* techreport 2989, C.E.P.R. Discussion Papers.

Mortensen, D. T. & Pissarides, C. A. (1994). Job creation and job destruction in the theory of unemployment. *Review of Economic Studies*, 61(3), 397–415.

Sjogren, A. & Vikstrom, J. (2015). How long and how much? learning about the design of wage subsidies from policy changes and discontinuities. *Labour Economics*, 34(C), 127–137.

7 Appendices

7.1 Appendix 1: Proof of Lemma 1

Expression (4) can be rearranged as

$$E_{y|\varepsilon}\left[\Pi_t\left(y\right)\mid\varepsilon\right]=\frac{1}{r+\lambda}\left[E\left[y\mid\varepsilon\right]+\left(1-\delta\right)h-w+\delta\int_0^w\left(w-y\right)dF\left(y\mid\varepsilon\right)\right].$$

The derivative of this expression with respect to ε is given by

$$\frac{\partial E_{y|\varepsilon}\left[\Pi_t\left(y\right)\mid\varepsilon\right]}{\partial\varepsilon}=\frac{1}{r+\lambda}\left[\int_0^{+\infty}y\frac{\partial f\left(y\mid\varepsilon\right)}{\partial\varepsilon}dy+\delta\int_0^w\left(w-y\right)\frac{\partial f\left(y\mid\varepsilon\right)}{\partial\varepsilon}dy\right]. \tag{21}$$

The integrals of this expression can be rewritten as

$$\begin{aligned}\int_0^{+\infty}y\frac{\partial f\left(y\mid\varepsilon\right)}{\partial\varepsilon}dy &= \int_0^{+\infty}y\frac{\partial^2 F\left(y\mid\varepsilon\right)}{\partial\varepsilon\partial y}dy,\\ \int_0^w\left(w-y\right)\frac{\partial f\left(y\mid\varepsilon\right)}{\partial\varepsilon}dy &= \int_0^w\left(w-y\right)\frac{\partial^2 F\left(y\mid\varepsilon\right)}{\partial\varepsilon\partial y}dy.\end{aligned}$$

Integration by parts leads to

$$\begin{aligned}\int_0^{+\infty}y\frac{\partial^2 F\left(y\mid\varepsilon\right)}{\partial\varepsilon\partial y}dy &= \left[y\frac{\partial F\left(y\mid\varepsilon\right)}{\partial\varepsilon}\right]_0^{+\infty}-\int_0^{+\infty}\frac{\partial F\left(y\mid\varepsilon\right)}{\partial\varepsilon}dy,\\ \int_0^w\left(w-y\right)\frac{\partial^2 F\left(y\mid\varepsilon\right)}{\partial\varepsilon\partial y}dy &= \left[\left(w-y\right)\frac{\partial F\left(y\mid\varepsilon\right)}{\partial\varepsilon}\right]_0^w+\int_0^w\frac{\partial F\left(y\mid\varepsilon\right)}{\partial\varepsilon}dy,\end{aligned}$$

where

$$\left[y\frac{\partial F\left(y\mid\varepsilon\right)}{\partial\varepsilon}\right]_0^{+\infty}=\left[\left(w-y\right)\frac{\partial F\left(y\mid\varepsilon\right)}{\partial\varepsilon}\right]_0^w=0.$$

This is due to the fact that

$$\lim_{y\to+\infty}F\left(y\mid\varepsilon\right)=1,\forall\varepsilon,$$

which implies that

$$\lim_{y\to+\infty}\frac{\partial F\left(y\mid\varepsilon\right)}{\partial\varepsilon}=0.$$

Similarly, because

$$F\left(0\mid\varepsilon\right)=0,\forall\varepsilon,$$

we have

$$\frac{\partial F\left(0\mid\varepsilon\right)}{\partial\varepsilon}=0.$$

Substituting into equation (21) and rearranging we end up with

$$\begin{aligned}\frac{\partial E_{y|\varepsilon}\left[\Pi_t\left(y\right)\mid\varepsilon\right]}{\partial\varepsilon} &= \frac{1}{r+\lambda}\left[-\int_0^w\left(1-\delta\right)\frac{\partial F\left(y\mid\varepsilon\right)}{\partial\varepsilon}dy-\int_w^{+\infty}\frac{\partial F\left(y\mid\varepsilon\right)}{\partial\varepsilon}dy\right]>0\\ &\Longleftarrow \frac{\partial F\left(y\mid\varepsilon\right)}{\partial\varepsilon}<0,\forall y\in\mathbb{R}_+,\end{aligned}$$

by the assumption of first order stochastic dominance and given that $\delta=e^{-(r+\lambda)\Delta}<1$.

7.2 Appendix 2: The firing rate $\bar{F}$ is decreasing in the vacancy threshold $\tilde{\varepsilon}$.

Recalling that

$$\bar{F}(\tilde{\varepsilon}) \equiv E_{\varepsilon \geq \tilde{\varepsilon}}\left[F(w \mid \varepsilon)\right] = \frac{\int_{\tilde{\varepsilon}}^{+\infty} F(w \mid \varepsilon)\, dG(\varepsilon)}{1 - G(\tilde{\varepsilon})}.$$

Applying Leibniz's rule, we find

$$\begin{aligned}
\frac{\partial \bar{F}(\tilde{\varepsilon})}{\partial \tilde{\varepsilon}} &= \frac{-F(w \mid \tilde{\varepsilon})\, g(\tilde{\varepsilon})\,(1 - G(\tilde{\varepsilon})) + g(\tilde{\varepsilon}) \int_{\tilde{\varepsilon}}^{+\infty} F(w \mid \varepsilon)\, dG(\varepsilon)}{(1 - G(\tilde{\varepsilon}))^2} < 0 \\
\iff & \quad E\left[F(w \mid \varepsilon \geq \tilde{\varepsilon})\right] < F(w \mid \tilde{\varepsilon}),
\end{aligned}$$

which is implied by first order stochastic dominance.

7.3 Appendix 3: Proof of Proposition 1

The first point of the Proposition states that $d\tilde{\varepsilon}/dh < 0$. This derivative can be found by applying the implicit function theorem to condition (5). Based on expression (4), we obtain

$$\frac{\partial \tilde{\varepsilon}}{\partial h} = -\frac{1-\delta}{r+\lambda} \left(\frac{\partial E_{y|\varepsilon}\left[\Pi_t(y) \mid \tilde{\varepsilon}\right]}{\partial \varepsilon} \right)^{-1} < 0,$$

by Lemma 1.

The proof of the second point is as follows. We apply the implicit function theorem to the free entry condition (6):

$$\frac{d\theta}{dh} = -\frac{d\Pi_J}{dh} \left(\frac{d\Pi_J}{d\theta} \right)^{-1},$$

where

$$\frac{d\Pi_J}{d\theta} = \left[\frac{\partial}{\partial \theta} \left(\frac{m(\theta)}{r + m(\theta)} \right) \right] m'(\theta) \int_{\tilde{\varepsilon}}^{+\infty} E\left[\Pi_t(y, h) \mid \varepsilon\right] dG(\varepsilon) < 0.$$

Therefore,

$$\frac{d\theta}{dh} > 0 \iff \frac{d\Pi_J}{dh} = \frac{\partial \Pi_J}{\partial h} + \frac{\partial \Pi_J}{\partial \tilde{\varepsilon}} \frac{\partial \tilde{\varepsilon}}{\partial h} > 0.$$

By Leibniz's rule and the definition of the vacancy threshold (5),

$$\frac{\partial \Pi_J}{\partial \tilde{\varepsilon}} = -\frac{m(\theta)}{r + m(\theta)} E_{y|\varepsilon}\left[\Pi_t(y) \mid \tilde{\varepsilon}\right] = 0.$$

It follows that

$$\frac{d\theta}{dh} > 0 \iff \frac{\partial \Pi_J}{\partial h} > 0,$$

where

$$\begin{aligned}
\frac{\partial \Pi_J}{\partial h} &= \frac{m(\theta)}{r + m(\theta)} \int_{\tilde{\varepsilon}}^{+\infty} \frac{\partial E_{y|\varepsilon}\left[\Pi_t(y) \mid \varepsilon\right]}{\partial h} dG(\varepsilon) \\
&= \frac{m(\theta)}{r + m(\theta)} (1 - G(\tilde{\varepsilon})) \frac{1-\delta}{r+\lambda} > 0,
\end{aligned}$$

where use has been made of equation (4).

7.4 Appendix 4: Proof of Proposition 4

By combining the flow equations (8, 9, 10), we get that

$$\ell_t = \frac{\Delta}{\lambda\Delta + e^{-\lambda\Delta}} \theta m(\theta) u, \tag{22}$$

$$\ell_p = \frac{1}{\lambda} \frac{e^{-\lambda\Delta}}{\lambda\Delta + e^{-\lambda\Delta}} \left(1 - \bar{F}\right) \theta m(\theta) u, \tag{23}$$

and that, as a result,

$$\frac{\ell_t}{\ell_t + \ell_p} = \frac{1}{1 + \left(1 - \bar{F}\right) \frac{e^{-\lambda\Delta}}{\lambda\Delta}}.$$

Because $\partial\bar{F}/\partial\tilde{\varepsilon} < 0$ (see Appendix 2) and $d\tilde{\varepsilon}/dh < 0$, by Proposition 1, one can see that $d\left(\ell_t/\left(\ell_t + \ell_p\right)\right)/dh > 0$.

In this Appendix, we also show that the mass of permanent jobs ℓ_p can be written as

$$\ell_p = \rho m(\theta) J \int_{\tilde{\varepsilon}}^{+\infty} \left(1 - F(w \mid \varepsilon)\right) dG(\varepsilon),$$

where

$$\rho = \frac{1}{\lambda} \frac{e^{-\lambda\Delta}}{\lambda\Delta + e^{-\lambda\Delta}}.$$

In light of (23), we see that we need to show that

$$\left(1 - \bar{F}\right) \theta m(\theta) u = m(\theta) J \int_{\tilde{\varepsilon}}^{+\infty} \left(1 - F(w \mid \varepsilon)\right) dG(\varepsilon). \tag{24}$$

Let us first highlight the following identity $\theta m(\theta) u = m(\theta) v$, which arises from the fact that the rate of match formation can be measured either from the viewpoint of workers (the left hand side), or from the viewpoint of employers (the right hand side). Also, by definition, the mass of vacancies and the firing rate are respectively given by

$$v \equiv J\left(1 - G(\tilde{\varepsilon})\right),$$

$$\bar{F} \equiv E_{\varepsilon \geq \tilde{\varepsilon}}\left[F(w \mid \varepsilon)\right] = \frac{\int_{\tilde{\varepsilon}}^{+\infty} F(w \mid \varepsilon) dG(\varepsilon)}{1 - G(\tilde{\varepsilon})}.$$

Combining these expressions shows that equation (24) holds. Indeed,

$$\int_{\tilde{\varepsilon}}^{+\infty} \left(1 - F(w \mid \varepsilon)\right) dG(\varepsilon) = \left(1 - G(\tilde{\varepsilon})\right)\left(1 - E_{\varepsilon \geq \tilde{\varepsilon}}\left[F(w \mid \varepsilon)\right]\right).$$

7.5 Appendix 5: Details of the model to be calibrated

The main task consists in translating and solving the model in discrete time. One period is normalized to 1 year in the calibration.

The first step of the model's resolution consists in finding the vacancy threshold $\tilde{\varepsilon}$. To this end, we calculate the asset value of a temporary job for each possible value of the signal ε:

The asset value of a permanent job writes

$$\Pi_p(y) = (y - w) + \delta(y - w) + \delta^2(y - w) + ... = \frac{1}{1 - \delta}(y - w), \tag{25}$$

where

$$\delta \equiv \frac{1 - \lambda}{1 + r},$$

with λ the probability that the job be destroyed for exogenous reasons during one period of time and $1/(1+r)$ is the discount factor. We assume that $\Delta = 2$, meaning that the fixed term contract lasts for 2 periods of time. As in the analytical model, this also corresponds to the duration of the subsidized period. The asset value of a temporary job is thus given by

$$\Pi_t(y) = (y - w + h) + \delta(y - w + h) + \delta^2 \max\{\Pi_p(y), 0\}.$$

Making use of equation (25), one obtains

$$\begin{aligned}\Pi_t(y) &= (1+\delta)(y - w + h), \text{ if } y \in [0, w) \\ &= (1+\delta) h + \frac{1}{1-\delta}(y - w), \text{ otherwise.}\end{aligned}$$

Taking the expectation over the productivity level conditional on the job's type,

$$E_{y|\varepsilon}[\Pi_t(y)] = (1+\delta) h + (1+\delta)\int_0^w (y - w)\, dF(y \mid \varepsilon) + \frac{1}{1-\delta}\int_w^{+\infty}(y - w)\, dF(y \mid \varepsilon).$$

Under our assumption that $y \mid \varepsilon \sim U\left[\varepsilon - (1-\alpha)\frac{\bar{y}}{2}, \varepsilon + (1-\alpha)\frac{\bar{y}}{2}\right]$, we have

$$\begin{aligned}F(w \mid \varepsilon) &= 1, \text{ if } y_H(\varepsilon) < w \iff \varepsilon < w - (1-\alpha)\frac{\bar{y}}{2}, \\ &= \frac{w - y_L(\varepsilon)}{(1-\alpha)\bar{y}}, \text{ if } y_L(\varepsilon) \leq w \leq y_H(\varepsilon) \iff w - (1-\alpha)\frac{\bar{y}}{2} \leq \varepsilon \leq w + r + (1-\alpha)\frac{\bar{y}}{2}, \\ &= 0, \text{ if } w < y_L(\varepsilon) \iff \varepsilon > w + (1-\alpha)\frac{\bar{y}}{2}.\end{aligned}$$

Recalling that the monotonicity of $E_{y|\varepsilon}[\Pi_t(y)]$ with respect to ε is guaranteed by Lemma 1, the threshold value $\tilde{\varepsilon}$ such that $E_{y|\varepsilon}[\Pi_t(y) \mid \tilde{\varepsilon}] = 0$ is found numerically.

In a second step, we derive the expression of the separation rate $\bar{F}(\tilde{\varepsilon})$:

$$\bar{F}(\tilde{\varepsilon}) \equiv E_{\varepsilon \geq \tilde{\varepsilon}}[F(w \mid \varepsilon)] = \frac{\int_{\tilde{\varepsilon}}^{\varepsilon_H} F(w \mid \varepsilon)\, d\varepsilon}{1 - G(\tilde{\varepsilon})} = \frac{\varepsilon_H - \varepsilon_L}{\varepsilon_H - \tilde{\varepsilon}}\int_{\tilde{\varepsilon}}^{\varepsilon_H} \max\{0, \min\{\phi(\varepsilon), 1\}\}\, d\varepsilon,$$

where

$$\phi(\varepsilon) = \frac{w - y_L(\varepsilon)}{(1-\alpha)\bar{y}}.$$

This function is calculated numerically.

Third, we calculate the probability to reach permanent employment (survival) of a newly employed temporary worker:

$$P(S > \Delta) = (1-\lambda)^2\left(1 - \bar{F}(\tilde{\varepsilon})\right).$$

This value is compared to its empirical counterpart in the calibration exercise.

We then calculate the value of the probability to fill a vacancy $m(\theta)$, which is compatible with the free entry condition. To this end, we write the free entry condition in discrete time as

$$\frac{m(\theta)}{1+r}\int_{\tilde{\varepsilon}}^{+\infty} E[\Pi_t(y, h) \mid \varepsilon]\, dG(\varepsilon) = \kappa \tag{26}$$

$$\iff m(\theta) = \frac{\kappa(1+r)}{\int_{\tilde{\varepsilon}}^{+\infty} E[\Pi_t(y, h) \mid \varepsilon]\, dG(\varepsilon)}.$$

The left hand side of condition (26) is the expected profit of drawing a job's type weighted by the discount factor and the probability to fill a vacancy. The right hand side is the lump sum cost. It can be seen that, in equilibrium, the probability to fill a vacancy is decreasing in the expected profit.

The next step consists in deriving market tightness by using the reciprocal function: $\theta = m^{-1}(m(\theta))$. Under the assumption that $\eta = 1/2$,

$$m(\theta) = \frac{A}{\sqrt{\theta}} \iff m^{-1}(x) = \left(\frac{A}{x}\right)^2.$$

Finally, let us re-formulate the Beveridge curve in discrete time. To this end, we equalize the flows into and out of the four following statuses: unemployment, first and second year temporary employment, and permanent employment. Let ℓ_{t1} and ℓ_{t2} denote the masses of first and second year temporary workers, respectively. Flow equations are as follows:

There is a constant mass of unemployed if and only if

$$\lambda(\ell_{t1} + \ell_{t2} + \ell_p) + (1-\lambda)\bar{F}\ell_{t2} = \theta m(\theta)u.$$

The mass of first year temporary workers is given by the flow out of unemployment:

$$\ell_{t1} = \theta m(\theta)u.$$

The mass of second year temporary workers is given by the number of surviving jobs in year 2:

$$\ell_{t2} = (1-\lambda)\theta m(\theta)u.$$

There is a constant mass of permanent jobs if and only if

$$(1-\lambda)\left(1-\bar{F}\right)\ell_{t2} = \lambda\ell_p.$$

Combining the flow equations and making use of the identity $\ell_p = 1 - u - \ell_{t1} - \ell_{t2}$, one obtains the Beveridge curve, which writes

$$u = \frac{\sigma}{\theta m(\theta) + \sigma},$$

where

$$\sigma \equiv \frac{\lambda}{1 - \bar{F}(1-\lambda)^2}.$$

7.6 Appendix 6: Calibration

Let us start by describing the transition probabilities as estimated from the CBSS as well as parameter values that are used in the baseline calibration. First, the probability of finding a job $\theta m(\theta)$ is estimated from transitions from unemployment to employment of the young (less than 25) in a one year time interval. We take the job finding rate averaged over 5 years for the most recent available data, which at the time of writing ranges from 2010-2011 to 2014-2015. This gives us an average rate of about 28%. Second, the exogenous destruction rate λ is proxied by the rate of job losses among workers who are employed for at least 3 consecutive years. Recall that in the model exogenous shocks are, indeed, the only source of permanent job destruction. Note that the structure of the data does not allow us to control for job to job transitions, which the model also ignores. Again, we calculate this rate for 5 time intervals ranging from 2008-2011 to 2012-2015 and take the average in order to approach steady state values. More precisely, we select young workers who were employed in years 1 to 3 and look at the proportion of them who end up unemployed in year 4. We find an average rate of 4.6%. Third, the calibration of the model requires us to have an estimate of the survival rate until permanent employment $P(S > \Delta)$. To be consistent with the assumption that $\Delta = 2$, we look at a 4 years window where young workers were initially unemployed and are employed in year 2. Among this cohort, we calculate the proportion of those who are still in employment, without interruption, in year 4. Like for the other estimates, we repeat this procedure 5 times from 2008-2011 to 2012-2015 and find an average survival rate of 66%. Finally, we make use of the expression of the survival rate until permanent employment of newly hired workers[10] to find our estimate of the separation rate $\bar{F}(\tilde{\varepsilon})$:

$$P(S > \Delta) = (1-\lambda)^2 \left(1 - \bar{F}(\tilde{\varepsilon})\right).$$

This gives us a value of about 28%.

Regarding other exogenous parameters, we arbitrarily assign a value of 2% to the interest rate, which determines the discount factor. This value corresponds to the inflation target of the central bank in the Euro zone. The highest possible productivity of a job $\bar{y}$ is normalized to 1. The proportion of information that the signal reveals α is fixed at 0.6 in the baseline calibration. Robustness exercises are executed with values of 0.5 and 0.7, respectively.

The remaining parameters are fixed so that the model replicates the above transition rates. In particular, the target value of $\tilde{\varepsilon}$ is such that $\bar{F}(\tilde{\varepsilon}) \simeq 0.28$. Recalling that $\tilde{\varepsilon}$ is determined by condition (5), which tells us that the expected profit of a vacancy is zero in equilibrium:

$$V(\tilde{\varepsilon}) = 0 \iff E_{y|\varepsilon}\left[\Pi_t(y) \mid \tilde{\varepsilon}\right] = 0,$$

where $E_{y|\varepsilon}\left[\Pi_t(y) \mid \tilde{\varepsilon}\right]$ depends on r, λ, w, h and on the parameters of the conditional distribution of productivity, as can be seen from (4). The hiring subsidy being fixed as a given fraction of the wage (see below), w is the only remaining degree of freedom. It is fixed so as to match the separation rate: $\bar{F}(\tilde{\varepsilon}) \simeq 0.28$. Two parameters remain to be calibrated, namely the total factor productivity of matching A and the fixed cost of a type draw κ. Their values are jointly chosen so as to match the job finding rate $\theta m(\theta) \simeq 0.28$ and the vacancy rate. According to the Eurostat definition, but unlike in our model, the latter is given by the number of vacancies over the sum of occupied positions and vacancies. Given that the spirit of this ratio is to measure the relative importance of unmet demand, we translate this measure as

$$\text{Vacancy rate} \equiv \frac{(1-m(\theta))\,v}{(1-m(\theta))\,v + (1-u)}.$$

In other words, we measure vacancies as the number of unmatched vacancies $(1-m(\theta))\,v$. The average vacancy rate over the period 2014-2018 is about 2.24% in Wallonia (source: Statbel). Recall that θ is ultimately fixed by κ such that the free entry condition is satisfied (6). On the one hand, the vacancy rate negatively depends on the filling rate $m(\theta)$, which is increasing in A and decreasing in θ. On the other hand, the job finding rate $\theta m(\theta)$ is increasing in A, but also in θ. It is therefore possible to find a vector (A, κ) such that the target values of $m(\theta)$ and $\theta m(\theta)$ are achieved. In other words, A and κ jointly determine the vacancy rate and the job finding rate.

Finally, we need to determine the amount of the hiring subsidy as a proportion of the wage. To this end and given the dynamic profile of the scheme, we calculate the average monthly subsidy over the subsidized

[10] See Appendix 5.

period for the scheme applying to the young (*Impulsion moins de 25 ans*), namely 396 euros.[11] As regards the wage, we take the gross monthly wage of the young in 2013, evaluated at 2017 prices and augmented of a rate of 33% of social security contributions, and obtain 3131 euros as our estimate of the labor cost.[12] We end up with a rate of subsidy of about 13%. This is the value that we assume in the baseline calibration.

[11] This gives $(24 * 500 + 6 * 250 + 6 * 125)/36 \simeq 396$ euros.

[12] We take 2017 as the reference year because the hiring subsidy scheme has been revised in that year. The gross salary is adapted to inflation given that in Belgium there is a mechanism of automatic wage indexation. Data come from EUROSTAT, Direction générale Statistique - Statistics Belgium, "Enquête sur la structure et la répartition des salaires".

Parameters that are constant across simulations	**Value**	**Source**
Job finding rate ($\theta m(\theta)$)	28%	Estimated from CBSS data
Exogenous destruction rate (λ)	4.6%	Estimated from CBSS data
Survival until permanent employment (P(S>Δ))	66%	Estimated from CBSS data
Separation rate (F bar)	28%	Given by λ and P(S>Δ) and a model's identity
Interest rate (r)	2%	Arbitrary
Supremum of the productivity distribution (y bar)	1	Normalization
Hiring subsidy as a proportion of the wage	13%	From the policy and Statistics Belgium
Vacancy rate	2.24%	Statistics Belgium (Eurostat definition)
Elasticity of the matching function (η)	0.5	Standard in the literature

Parameters that vary between simulations	**Value (baseline)**	**Source**
Informativeness of the signal (α)	0.6	Arbitrary (a sensitivity analysis is provided)
Wage (w)	0.47	Fixed so as to match the separation rate
Total factor productivity of matching (A)	0.45	A and κ are jointly determined so as to match
Fixed cost of a type draw (κ)	1150	the job finding rate and the vacancy rate

Table 1: Transition probabilities and parameter values used in the baseline calibration

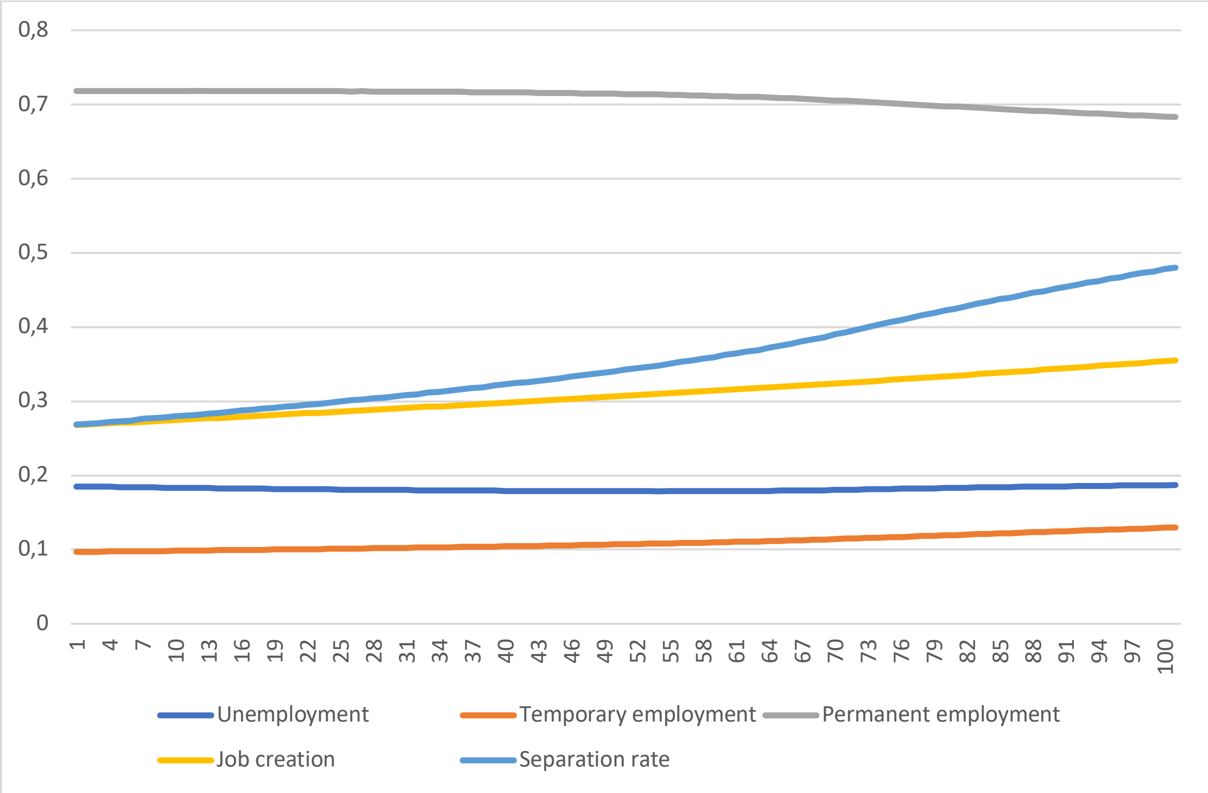

Figure 1 : Flows and stocks as a function of the hiring subsidy. Alpha = 0.5

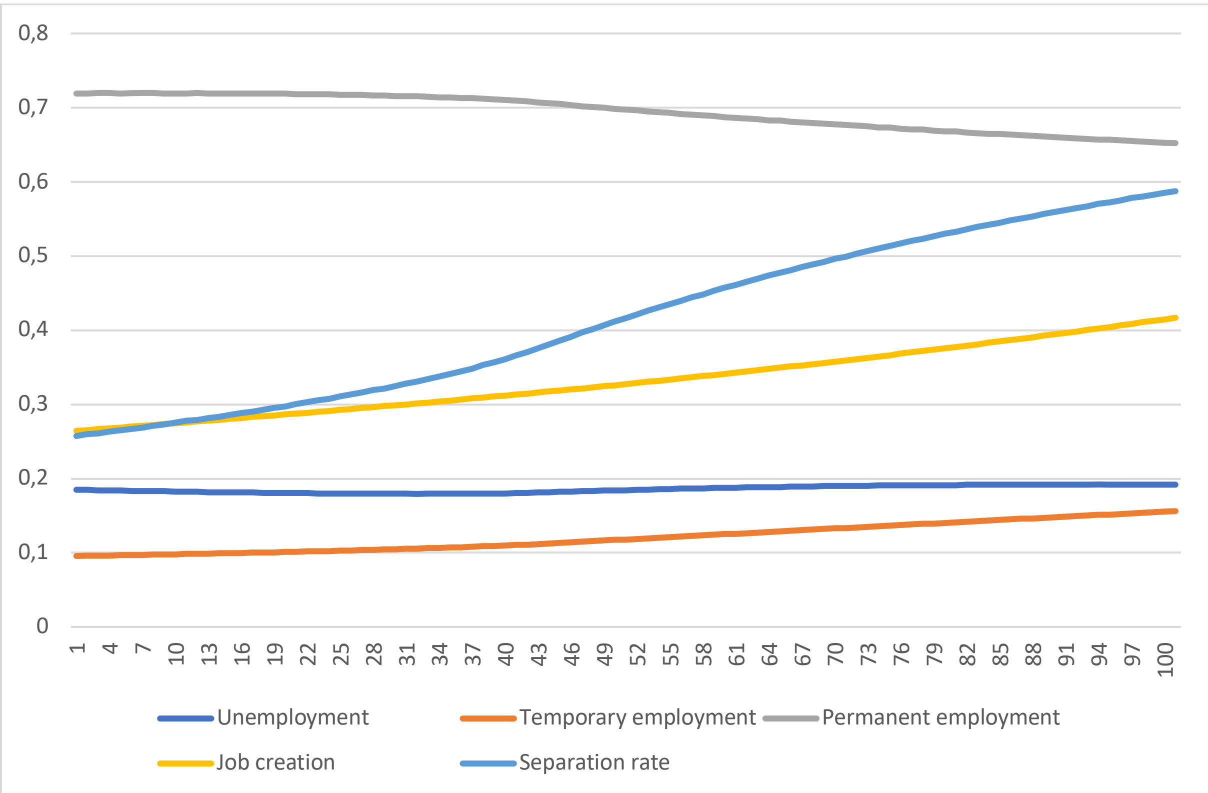

Figure 2 : Flows and stocks as a function of the hiring subsidy. Alpha = 0.6

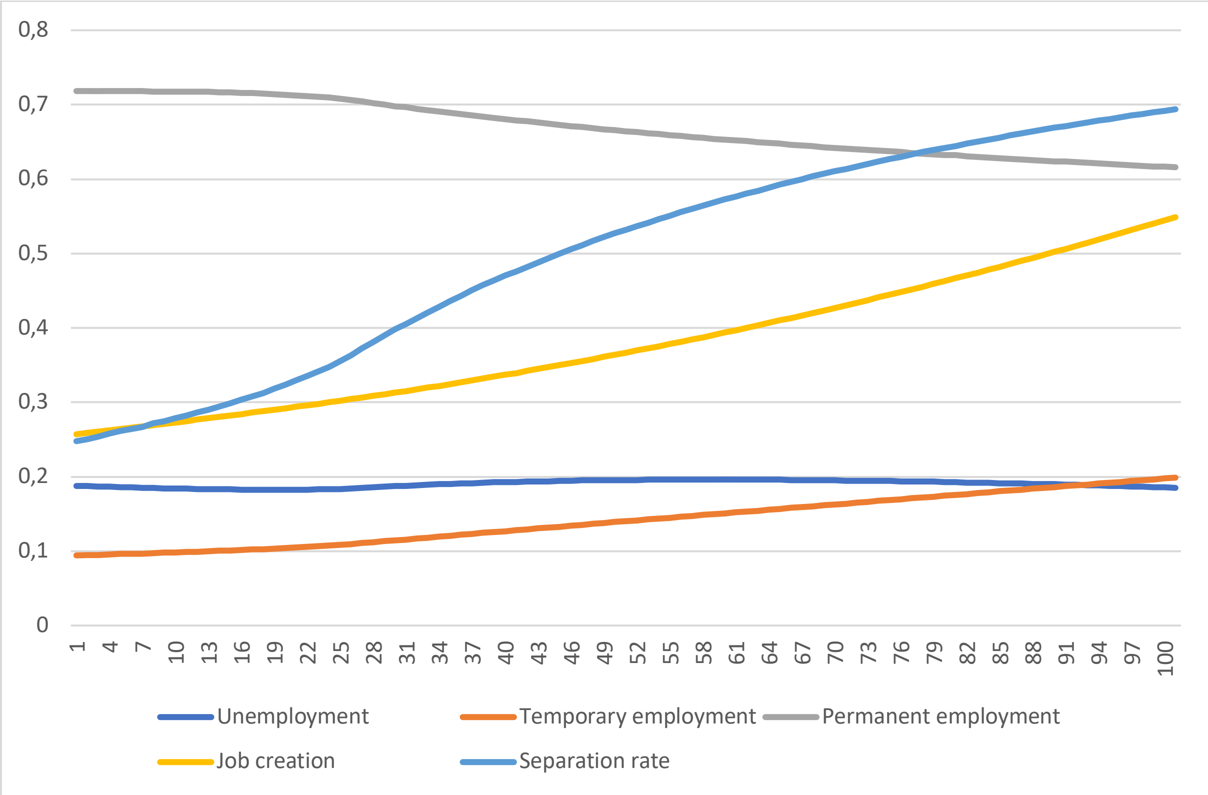

Figure 3 : Flows and stocks as a function of the hiring subsidy. Alpha = 0.7