

Optimal child care policies with heterogenous parents: Should I care or should I work?

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February 15, 2022

Abstract

Child care policies have significant effects on mothers' labor supply and on child development, which in turn affect social welfare. Yet, the objectives and forms of policy intervention are still under debate. We provide a normative analysis of the child care sector in which parents, who are heterogeneous in both their earnings potential and their quality as childminders, do not fully internalize the social benefits of child quality. The social planner designs the availability, price and quality of formal childcare and seeks to maximize the welfare of parents and children subject to the economy's budget constraint. At the first best, the planner equalizes consumption across households and quality across children. This allocation is decentralizable only if the social planner has complete information on parents' quality as childminders. Under incomplete information, the planner lacks the instruments to incentivize parents to make optimal use of formal child care and to redistribute resources across both dimensions of parent heterogeneity. Our second best analysis allows to characterize the optimal child care policy when governments pursue the twofold objective of producing the socially desirable level of child quality and of mitigating income inequalities.

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1 Introduction

Formal child care is at the intersection of important societal challenges, due to its impacts on women's labor market participation (Domeij and Klein (2013), Bauernschuster and Schlotter (2015), Bick (2016)) as well as on children's cognitive and social skills (Heckman et al. (2013), Boca et al. (2014)). As formal child care generates multi-generational welfare effects and externalities, there is no clearcut conclusion on how to optimally provide it. In the words of Blau (2003): "Child care policy can be used to facilitate employment of mothers and enhance the development of young children. The tension between these alternative goals ensures that debate and discussion of child care policy issues will continue for the foreseeable future. There is not a consensus on the goals of child care policy or on the means to achieve those goals." Parent heterogeneity makes the tension between these goals even more stringent. Indeed, should formal child care be provided in priority to highly skilled women, based on their high returns to labor market participation (Bauernschuster and Schlotter (2015))? In contrast, children from the least educated households benefit more from formal child care (Heckman et al. (2013)). To what extent do human capital externalities influence this tradeoff?

In this paper, we study the optimal design of public child care in an economy with parent heterogeneity, incomplete information and human capital externalities. More specifically, parents are heterogenous in their earnings potential and their ability to care for their child, which the social planner may not observe. Also, parents may not fully internalize the social benefits of child quality. We first show that the tradeoff between short-term economic efficiency and child quality varies across parents. Under complete information, the first best allocation can be decentralized, with child care price and parental allowance schemes playing an incentive and redistributive role, respectively. When the planner does not observe parental care quality, a tradeoff arises between the optimal production of child quality and the mitigation of income inequality between households.

1.1 The need for policy intervention

In this section, we review the literature which identifies various types of externalities related to child care. These externalities are the main justification for policy intervention, and are key ingredients to our model. They can be grouped into two categories, namely externalities linked to children (lack of internalization of returns to child care and inequality reduction) and externalities linked to parents (labour market effects and redistribution).

Formal and parental care are important inputs in the production of child quality (Heckman et al. (2013), Boca et al. (2014)). Beyond its private returns, such as lifetime earnings, child quality generates significant social returns, indirectly through redistribution, but also through other types of outcomes such as health and criminality. By shaping the use of formal and parental care, child care policies have the potential to foster the internalization of these social returns, as highlighted in our framework. It is also worth noting that, in addition to positive externalities, parents may also fail to fully internalize private benefits to child quality. For instance, Baland and Robinson (2000) shows that even though they may be fully altruistic, poor parents facing liquidity constraints may sub-optimally invest in human capital.

Heckman et al. (2013) shows that the effect of formal child care on cognitive and social skills is stronger among disadvantaged children. Our model accounts for parent's heterogeneity in their care quality as well as their earnings potential, which allows us to incorporate the potential of child care policies to reduce inequality

of opportunity among children.¹

Externalities on the current generation mainly pertain to parents' labor supply, which is part of our analysis. The provision of formal child care has the potential to foster parents', especially mothers' participation to the labour market (Bauernschuster and Schlotter (2015), Bick (2016)). Again, beyond the private benefits of increasing labour supply, formal child care generates positive externalities. Domeij and Klein (2013) shows that labor taxes particularly disincentivize the labour supply of parents of young children. Therefore, by reducing the distortion in the tradeoff between consumption and leisure, formal childcare has a positive impact on resource allocation efficiency and eventually on social welfare.²

Finally, increasing labor supply of mothers has the potential to reduce gender inequality. It is, for instance, largely acknowledged that mothers' career breaks impedes the accumulation of human capital and work experience, which has a long-lasting negative impact on earnings and professional achievements (Eckstein and Wolpin (1989), Blundell et al. (2016), Turon (2016)). Intra-household arrangements are affected by the availability of formal child care, and collective household models treat child care as a contribution to a household public good (Blundell et al. (2005), Cherchye et al. (2012)). While household allocation is generally assumed to be Pareto efficient, Basu (2006) proposes an extension of the collective model where Pareto weights are allowed to depend on labor supply decisions. Even though the initial labor supply decision is efficient and results from the initial balance of power, in a dynamic context the second earner ends up with a lower bargaining weight. In a collective model which incorporates divorce, Turon (2016) argues that the hazard of divorce might lead to inefficient ex post intra-household arrangements. If the burden of parental care is unevenly distributed within couples at the expense of the mother's labor supply, then the household fails to fully internalize her foregone labour market potential, which impacts the mother more heavily in the event of divorce. Although potentially very interesting, taking into account the impact of child care policies on household dynamics in the design of such policies is left for future research.

Our model accounts for externalities linked to child quality and labour supply in the presence of heterogeneous parents. The first best analysis identifies what types of parents should benefit from formal child care and with what intensity, so as to achieve optimal child quality and to mitigate income inequality via a social welfare function. This approach allows us to characterize the optimal supply of formal child care in terms of quality and price and to shed light on the desirable forms of policy intervention. The next sub-section refers to the scarce existing literature on the design of child care policies.

1.2 The form of policy intervention

The impact of policy intervention on parents' labor supply and child care decisions has been amply documented (See for instance recent evidence by Domeij and Klein (2013), Bauernschuster and Schlotter (2015), Bick (2016), Blundell et al. (2016)). However, to the best of our knowledge, contributions on the optimal design of public policies are scarce.

¹Another type of externality related to children, which is left out of our analysis, pertains to the impact of child care policies on fertility. Recent evidence from Germany is however mixed: Bauernschuster et al. (2016) find a significantly positive effect at the intensive margin, while Bick (2016) finds no significant effect.

²Formal childcare can also generate externalities on labour supply through peer effects. Nicoletti et al. (2016) indeed shows that mothers' labor supply is positively affected by the number of hours worked by other mothers in their family network. The local availability of child care services generates positive externalities on other mothers, even in the event where child care facilities are not directly available to them.

Blau (2003), who compares unconditional child care subsidies to subsidies conditional on quality, highlights conflicting objectives of child care policies in terms of child quality and labor force participation. Using a life-cycle model of labor supply and human capital accumulation, Blundell et al. (2016) estimates the impact of tax and welfare reforms on labor supply. They argue that tax credit has to be preferred to income support because it is less prone to moral hazard.

Domeij and Klein (2013) studies optimal taxation in the presence of children with linear taxes and child care subsidies. Due to the higher distortionary effect of the labor tax for parents of young children, they argue that the aggregate tax burden should vary over the life cycle, and in particular be lighter when children are young. This can be achieved either through child care subsidies or by making child care expenses tax deductible.

Our model also studies optimal tax and child care subsidies, but differs from Domeij and Klein (2013) in different respects. Most importantly, our paper places child quality as one of the key drivers of child care policies. In Domeij and Klein (2013), children appear as a constraint on parental time (justifying the use of formal child care as in our model), but child quality is not valued per se, neither by the parents nor by the planner. Also, parental care is treated as leisure in their model, while we consider it as an input in the production of child quality. It results that the source of inefficiency in Domeij and Klein (2013) is different from ours, and stems from preferences (distorted labor supply) while we focus on externalities in the production of child quality. Also, Domeij and Klein (2013) considers income taxation over the life course, while our approach is not dynamic but allows for non-linear policy instruments. Finally, like ours, their model accounts for heterogeneity among parents, but we add and treat the case of incomplete information, where parents' characteristics are not fully observable by the planner.

Glomm and Meier (2016) also propose an analysis of the optimal form of childcare intervention. The main message of this paper is close to Domeij and Klein (2013)'s conclusion in that it highlights the role that childcare subsidies may play to offset the distortionary impact of labor taxes. Like us, they study the shape of optimal subsidy as a function of the wage rate of the primary care giver. The main difference with our setting is that they do not consider inequality reduction as part of the social planner's objectives. In the absence of a redistributive motive and of externalities related to children, they obtain a childcare subsidy which is upward sloping in the parent's wage so as to compensate for the distortionary labor tax. This slope decreases when parents imperfectly internalize child outcomes. As compared to Glomm and Meier (2016)'s contribution, we focus throughout on externalities related to children and study how to resolve them. Also, we want to see how a planner facing heterogeneous parents can or cannot combine efficiency (the production of child quality) and redistributive objectives both under complete and incomplete information.

This paper is organized as follows. In Section 2, we introduce the main assumptions of the model: we describe the population of parents, their types and preferences and the technology, namely the child quality production function and the cost structure of the formal childcare sector. Section 3 introduces the social welfare function and contains the first best analysis. The second best problem, where parents make their own decision on labor supply and child care options, is solved in Section 4. We first explore the parents' reaction to the planner's set of policy variables, which includes labor tax instruments, formal child care subsidies (prices) and parental allowances. Second, we explore the case of complete information, where both the parent's wage and his/her parental care quality is assumed observable. Finally, we tackle the more realistic situation where parental care quality is private information. Section 5 concludes.

2 The model

Let us consider an economy composed of a continuum of households of measure one. Each household is composed of one parent p and one child k , who are both endowed with one unit of time.³ During this unit of time, the child needs to be cared for, while the parent can either work and/or rear his/her child. Parents are heterogeneously endowed in human capital, captured by a wage rate w . Parents can use either parental or formal child care. The set of child care options is $\{p, f\}$, where p , and f respectively define parental and formal child care.

Parents differ in human capital / wage $w \in [0, w^H]$ per unit of time, but also in parental care quality $q_p \in [0, q_p^H]$, which represents other parental skills specifically affecting child development. These two parental characteristics may be positively correlated, but our model allows for any type of statistical relationship between the two by setting a given joint distribution $h(w, q_p)$, which is left unspecified.

Given this variety of services, children receive a care quality

$$q = t_f q_f + (1 - t_f) q_p, \quad (1)$$

where $t_f \in [0, 1]$ is the fraction of time spent in formal child care. The total need for formal child care services is therefore $T_f = \iint t_f(w, q_p) h(w, q_p) dq_p dw$.

There are n formal child care providers which provide the same quality level q_f and share the demand for formal child care equally. The demand (in time) per formal child care provider is written $\tau_f = T_f/n$. These providers face the same cost function, which depends on the number of registered children and quality: $\kappa + C(\tau_f, q_f)$, with $C'_\tau > 0$, $C''_{\tau\tau} \geq 0$, $C'_q > 0$, $C''_{qq} \geq 0$ and $C''_{\tau q} > 0$ and where κ is a fixed cost per child care facility. For instance, C'_τ should be interpreted as the cost of increasing attendance by 1 child during 1 unit of time.

Finally, parents have homogeneous preferences. Their utility function is defined over their own consumption c of the numeraire good, per unit of time, and the child care quality received by their child, q . Utility is written

$$U(c, q) = u(c) + \gamma v(q), \quad (2)$$

where $u' > 0$, $u'' \leq 0$, $v' > 0$ and $v'' \leq 0$.⁴ This formulation can be interpreted in the following way. $v(q)$ can be seen as the child's current and/or future wellbeing as a function of the quality of early childhood education.⁵ In accordance with this interpretation, $\gamma \in [0, 1]$ can be directly interpreted as a coefficient of parental altruism.

We first study the first best problem, where the planner is able to decide on the characteristics of the formal child care sector, including the quality and the number of child care facilities and to enforce the children time allocation between formal and parental care. In a second step, we tackle the second best problem, where parents are free to choose their preferred mix of child care options. In this setting, the planner is only allowed

³This normalization does not affect our results. The reference period can be a day or a month.

⁴In order to avoid pathological cases in the resolution of the model, we make the following additional mild assumptions:

$$\begin{aligned} v'''(q) &\geq 0; -\frac{v''(q)}{v'(q)} \leq \frac{1}{q_p^H}, \forall q; \\ -\frac{u''(c)}{u'(c)} &\leq \frac{1}{w^H}. \end{aligned}$$

See Appendices for more details.

⁵Heckman et al. (2013) have shown that child development was indeed strongly affected by early childhood interventions.

to design public policies, including the tax scheme and other instruments pertaining to the child care policy, so as to influence parents' choices. We explore the second best under various assumptions regarding the observability of the parent's type (w, q_f) .

3 The first best

At the first best, the planner perfectly observes parents' characteristics (w, q_p) , and controls the behavior of all agents, i.e. parents and formal providers. As a result, the planner designs the number and quality of formal providers (n, q_f) , determines the consumption levels $c(w, q_p)$ and child care uses $t_f(w, q_p)$ of all households based on their characteristics. The planner sets these variables in order to maximize social welfare, captured by the social welfare function:

$$W = \iint [U(c, q) + \delta v(q)] h(w, q_p) dq_p dw, \quad (3)$$

where $\delta \in \mathbb{R}_+$ is the (extra) Pareto weight that the planner attributes to children, considering that the Pareto weight on parents is normalized to 1. Notice that, owing to the altruism of parents, the wellbeing of children is already incorporated in $U(c, q)$. We allow for an additional weight δ in order to represent cases where it is considered that parents imperfectly internalize the wellbeing of children. δ is therefore indeed a measure of the extra weight attributed to children. Recall that the main goal of this model is to identify optimal child care policies as a function of different policy objectives. In this respect, a higher δ will capture the government's willingness to improve the wellbeing of future generations through a higher (and a more equal distribution of) child quality.⁶ Making use of the parents' utility function (2), we can write the welfare function while disentangling the two components of social welfare, namely the wellbeing that adults derive from private consumption and children wellbeing:

$$W = W_p + \rho W_k, \quad (4)$$

where

$$W_p = \iint u(c(w, q_p)) h(w, q_p) dq_p dw, \quad (5)$$

$$W_k = \iint v(q(w, q_p)) h(w, q_p) dq_p dw, \quad (6)$$

$$\rho = \gamma + \delta,$$

$$q = t_f(w, q_p) q_f + (1 - t_f(w, q_p)) q_p.$$

The planner must define its policy while satisfying a budget constraint:

$$B = \iint (t_f(w, q_p) w - c(w, q_p)) h(w, q_p) dw dq_p + G - F \geq 0. \quad (7)$$

The first term gives the aggregate surplus, or deficit, of parental income over parents' private consumption. Notice that the opportunity cost of parental care, namely $(1 - t_f(w, q_p)) w$ for a household of type (w, q_p) is counted in this term given that w is the parent's full income per unit of time. The second term, $G \in \mathbb{R}_+$,

⁶Because $v(\cdot)$ is concave, the planner technically cares about equity in the distribution of child quality across households.

is the government's endowment allocated to child care policies and is exogenously given. The last term, F , measures the total cost of formal child care:

$$F \equiv n \left[\kappa + C \left(\frac{1}{n} \iint t_f(w, q_p) h(w, q_p) dq_p dw, q_f \right) \right]. \quad (8)$$

Finally, let us denote by λ the opportunity cost of public funds.⁷

Proposition 1 *The first best allocation* $\left(c^{FB}(w, q_p), t_f^{FB}(w, q_p), q_f^{FB}, n^{FB} \right)$

- *At the first best, the level of consumption of the numeraire $c^{FB}(w, q_p)$ is equalized between parents and such that*

$$u'(c^{FB}) = \lambda. \quad (9)$$

- *The first best time use of formal child care $t_f^{FB}(w, q_p)$ is such that*

$$\begin{aligned} t_f^{FB}(w, q_p) &= 1 \text{ if } q_p \leq \tilde{q}_p^{1,FB}(w), \\ &= t_f^{**}(w, q_p) \in (0, 1) \text{ if } \tilde{q}_p^{1,FB}(w) < q_p < \tilde{q}_p^{0,FB}(w), \\ &= 0 \text{ if } q_p \geq \tilde{q}_p^{0,FB}(w), \end{aligned} \quad (10)$$

with $\partial t_f^{**}/\partial w > 0$; $\partial t_f^{**}/\partial q_p < 0$ and where $t_f^{**}(w, q_p)$, $\tilde{q}_p^{1,FB}$ and $\tilde{q}_p^{0,FB}$ are respectively such that

$$\begin{aligned} \rho v'(q(t_f^{**}, q_p))(q_f - q_p) &= \lambda(C'_\tau - w), \\ \rho v'(q_f)(q_f - \tilde{q}_p^{1,FB}) &= \lambda(C'_\tau - w), \\ \rho v'(\tilde{q}_p^{0,FB})(q_f - \tilde{q}_p^{0,FB}) &= \lambda(C'_\tau - w). \end{aligned}$$

- *The first best formal child care quality q_f^{FB} is such that*

$$\rho \iint t_f^{FB}(w, q_p) v'(q) h(w, q_p) dq_p dw = \lambda n C'_q \quad (11)$$

- *The first best number of providers n^{FB} is such that*

$$C'_\tau(\tau_f, q_f) = \frac{\kappa + C(\tau_f, q_f)}{\tau_f} \quad (12)$$

Proof. Provided in Appendix 1. ■

Proposition 1 describes the first best allocation of consumption and time as well as the optimal structure of the child care sector.

We first note that the first best level of consumption, which is equalized between parents, is such that the marginal utility of consumption is equal to the opportunity cost of public funds.

Second, Proposition 1 provides the first best time use pattern, which is illustrated in Figure 1 in the space of household types (w, q_p) .

FIGURE 1 HERE

As can be seen from Figure 1, we can separate the space of parents' types into 4 quadrants. Parents who lie in the upper left quadrant provide a higher level of quality than formal child care ($q_p > q_f$). At the same time, the care they provide is less costly than formal child care because the opportunity cost of their time is

⁷ λ is the Lagrange multiplier of the budget constraint in the planner's optimization problem. Details are provided in appendix.

lower than the marginal cost of time use of formal child care ($w < C'_\tau$). Quite naturally, the planner assigns them to a corner solution $t_f^{FB} = 0$. The opposite can be said about parents lying in the lower right quadrant. These parents provide a care of lower quality ($q_p < q_f$) at a higher cost ($w > C'_\tau$), because of their high value on the labor market. They are assigned to $t_f^{FB} = 1$. For the two other categories of parents, the planner is facing a tradeoff. As compared to formal child care, parents in the lower left quadrant provide a lower quality of care at a lower cost. The opposite can be said about parents in the upper right quadrant. Interior solutions for time use will be found among these two categories only. In the remainder of the text, we will refer to parents who are interior in time use as low types if they have $q_p < q_f$ and as high types otherwise. As one would expect, other things equal, time use of formal child care among "interior parents" is decreasing in parental care quality and increasing in the parent's wage. Indeed, from a social welfare point of view, the marginal benefit of parental care is increasing in its quality and the marginal cost is increasing in the wage, which measures the opportunity cost of parents' time.

Regarding the characteristics of the formal child care sector, the third point defines the first best level of formal child care quality. Condition (11) states that the marginal benefit of formal child care quality should be equal to its marginal cost. The former is the product of the total marginal increase in child wellbeing $\iint t_f^{FB}(w, q_p) v'(q) h(w, q_p) dq_p dw$, which depends on children's exposure to formal child care t_f^{FB} , and the total (parent and planner) weight on children wellbeing ρ . The latter is given by $\lambda n C'_q$. It takes account of the shadow price of public funds λ and of the cost of increasing q_f marginally over all child care facilities $n C'_q$.

Fourth and finally, condition (12) tells us that the optimal number of child care facilities is such that the marginal cost and the average cost of time use are equalized, so as to minimize total cost.

4 The second best

In this section, we study the second best problem, where parents have the freedom to choose their preferred child care option. The planner still has perfect regulatory power over the quality and price of formal child care providers. Furthermore, we assume that the planner has at its disposal a series of instruments that influence the allocation of consumption and time use. The next subsection aims at analyzing parents' reaction to these policy variables. In the remainder of the paper, we then consider two versions of the second best. In the first one, the planner observes the full type of parents (w, q_p) (complete information) and is allowed to condition its intervention on the full type. The second version is more realistic as it is considered that the quality of parental care q_p is private information (incomplete information). Accordingly, the planner can only condition its intervention on the parent's wage w .

4.1 Parents' reaction to policy instruments

The planner maximizes social welfare (4) by defining its policy intervention. Adopting an exhaustive view of policy instruments, two broad categories of interventions can be considered. The first one consists of fiscal instruments, such as lump sum transfers and labor taxes. The second category covers instruments that specifically belong to the child care policy and includes (1) the parental allowance, which is granted to parents in proportion of the time spent at child rearing and (2) the formal child care price scheme. Fiscal and child care policies have two types of effects. They allow the planner to redistribute income / consumption and

to provide incentives regarding time allocation decisions, which at the same time determine parental labor supply and formal child care use.

We show that, in this model, fiscal instruments are redundant, so that we can focus on child care policies in isolation.

Regarding the fiscal policy, let $r(w, q_p)$ and $\theta(w, q_p)$ denote the lump sum transfer and a proportional labor tax, respectively. Regarding the child care policy, we introduce $w_0(w, q_p)$ as the full time equivalent parental allowance and $p_f(w, q_p)$ as the full time price of formal child care charged to a parent of type (w, q_p) . Notice that, in cases where the full type (w, q_p) is observable, these variables can depend on it. In order to save on notation, we however do not indicate it systematically. Taking account of these instruments, the consumption level of a parent of type (w, q_p) writes

$$c(w, q_p) = r + (1 - t_f)w_0 + t_f[(1 - \theta)w - p_f]. \quad (13)$$

Taking policies as given, parents choose their preferred time allocation, thereby determining labor supply and formal child care use. These choices are based on their utility (2), where consumption and child quality are given by equations (13) and (1), respectively. Let $t_f^{BR}(r, \theta, w_0, p_f; w, q_p)$ stand for the parent's best response to the planner's policy instruments.

Lemma 1 *Parents' time allocation:* *For a given vector of policy (r, θ, w_0, p_f) , parents' labor supply and formal child care use t_f^{BR} is such that*

$$\begin{aligned} t_f^{BR}(w, q_p) &= 1 \text{ if } q_p \leq \tilde{q}_p^{1, BR}(w), \\ &= t_f^*(w, q_p) \in (0, 1) \text{ if } \tilde{q}_p^{1, BR}(w) < q_p < \tilde{q}_p^{0, BR}(w), \\ &= 0 \text{ if } q_p \geq \tilde{q}_p^{0, BR}(w), \end{aligned}$$

where $t_f^*(w, q_p)$, $\tilde{q}_p^{1, BR}$ and $\tilde{q}_p^{0, BR}$ are respectively such that

$$\begin{aligned} \gamma v'(q(t_f^*, q_p))(q_f - q_p) &= -u'(c)[(1 - \theta)w - p_f - w_0], \\ \gamma v'(q_f)(q_f - \tilde{q}_p^{1, BR}) &= -u'(c)[(1 - \theta)w - p_f - w_0], \\ \gamma v'(\tilde{q}_p^{0, BR})(q_f - \tilde{q}_p^{0, BR}) &= -u'(c)[(1 - \theta)w - p_f - w_0]. \end{aligned} \quad (14)$$

Proof. This can be easily seen by looking at the first derivative of parents' utility (2) with respect to time use:

$$\frac{\partial U}{\partial t_f} = -u'(c)\pi_f + \gamma v'(q) \frac{\partial q}{\partial t_f},$$

where

$$\begin{aligned} \pi_f &\equiv -\frac{\partial c}{\partial t_f} = p_f + w_0 - (1 - \theta)w, \\ \frac{\partial q}{\partial t_f} &= q_f - q_p, \end{aligned}$$

and by taking account of corner solutions. ■

The following proposition studies the marginal impact of the different policy parameters on the parents' time use decision. Acknowledging that parents at a corner solution do not modify their decision when policy parameters vary at the margin, we can focus on the reaction of parents at an interior solution (with q_p

$\in [\tilde{q}_p^{1,BR}(w), \tilde{q}_p^{0,BR}(w)]$), for whom condition (14) is satisfied. Making use of the implicit function theorem, we find that a parent's reaction to any given policy parameter $\psi \in (r, \theta, w_0, p_f)$ has the following form

$$\frac{\partial t_f^*}{\partial \psi} = \Delta_I \frac{\partial c}{\partial \psi} + \Delta_S \frac{\partial \pi_f}{\partial \psi}, \quad (15)$$

where

$$\Delta_I \equiv \left. \frac{\partial t_f^*}{\partial c} \right|_{\pi_f \text{ cst}} = u''(c) \pi_f \left(\frac{\partial^2 U}{\partial t_f^2} \right)^{-1} > 0 \iff \pi_f > 0, \quad (16)$$

$$\Delta_S \equiv \left. \frac{\partial t_f^*}{\partial \pi_f} \right|_{u'(c) \text{ cst}} = u'(c) \left(\frac{\partial^2 U}{\partial t_f^2} \right)^{-1} < 0. \quad (17)$$

The total impact of any parameter ψ on parents' decision is thus composed of two effects. On the one hand, the first term on the right hand side of (15) measures the impact of an increase in consumption, directly caused by ψ , on labor supply and child care decisions. Δ_I is the fraction of the change in t_f^* caused by an increase in c by one unit, everything else constant. Because consumption and income are equal in the context of this paper, we will call Δ_I the income effect. On the other hand, the second term on the right hand side of (15) captures the effect of a change in $\partial c / \partial t_f$ caused by ψ on parents' decision. In particular, Δ_S measures the fraction of the change in t_f^* caused by an increase in $\partial c / \partial t_f$ by one unit, for a constant level of consumption c . Because $\partial c / \partial t_f$ measures the opportunity cost of the time spent at home rather than on the labor market, we will call Δ_S the substitution effect.

Proposition 2 *Impact of the planner's policies on parental choices: Formal child care use is*

1. increasing in the lump sum transfer for low types and decreasing for high types: $\partial t_f^* / \partial r = \Delta_I \leq 0 \iff q_p \geq q_f$,
2. decreasing in the labor tax: $\partial t_f^* / \partial \theta = w(\Delta_S - t_f \Delta_I) < 0$,
3. decreasing in the parental allowance: $\partial t_f^* / \partial w_0 = (1 - t_f) \Delta_I + \Delta_S < 0$,
4. decreasing in the price of formal child care: $\partial t_f^* / \partial p_f = \Delta_S - t_f \Delta_I < 0$.

Proof. Provided in Appendix 2. ■

Proposition 2 characterizes the parent's reaction to the different policy instruments that are at the planner's disposal. The first effect, which pertains to the lump sum transfer, is a pure income effect (Δ_I) since it does not affect prices. The sign of the income effect depends on the parent's type. All types of parents have in common the willingness to increase their child's quality as their income increases. To do so, low types for whom $q_p < q_f$ need to increase their use of formal child care and hence their labor supply. On the contrary, high types need to increase their supply of parental care, which is of higher quality $q_p \geq q_f$, and hence to decrease their labor supply.

Second, as expected, the labor tax discourages the use of formal child care. A higher labor tax decreases the net wage and is the exact equivalent of an increase in the price of formal child care, as can be seen by comparing points 2 and 4 of the Proposition. There are 2 effects. On the one hand, the substitution effect Δ_S is negative: a higher labor tax entails a lower net wage and results in a reduction in labor supply. Equivalently, a higher price of formal child care reduces demand for formal child care. On the other hand,

there is an income effect $t_f \Delta_I$, which is proportional to the labor supply (and the use of formal child care) t_f . Following an increase of either the labor tax or the price of formal child care, parents, who become poorer, react by reducing the target level (their "consumption") of their child's quality. Low types therefore decrease their use of formal child care, which reinforces the substitution effect. On the contrary, this effect mitigates the substitution effect for high types. Appendix 2 shows that the substitution effect still dominates the income effect for high types.

Third, an increase in the parental allowance naturally decreases the use of formal child care and parents' labor supply. This is the direction of the substitution effect. Again, the income effect $(1 - t_f) \Delta_I$, which is proportional to the time spent at home, has opposite effects on parents of low and high types. High types tend to decrease their use of formal child care and low types tend to increase it. The total effect is unambiguously negative for high types, while the (negative) substitution effect dominates for low types, as shown in Appendix 2.

Finally, it can be seen that, out of the 4 instruments, which can be either classified as tax instruments (r, θ) or as child care policies (w_0, p_f) , 2 are redundant. As already mentioned, we observe that the effects of the labor tax and of the price of formal child care are equivalent (see points 2 and 4 of Proposition 2). Both affect the price of the time spent on the labor market in exactly the same manner. More generally, a careful inspection of Proposition 2 reveals that only two types of effects are involved, namely the substitution effect Δ_S and the income effect Δ_I . Only 2 instruments are needed to combine them in two different ways.⁸ For instance, the parental allowance and the price of formal child care affect relative prices the same way, as they increase the relative value of time spent at home (decrease the relative value of time spent on the labor market). They however have opposite income effects. This appears more clearly in the next subsection, which shows that the planner can replicate the First Best allocation by making use of child care policies (w_0, p_f) only. Since only 2 instruments produce orthogonal effects, we focus on child care policies (w_0, p_f) and neglect tax instruments (r, θ) in the remainder of the analysis.

4.2 The second best under complete information

Let us now study the second best problem. In a relevant second best setting, the planner only observes wages w , but does not know the actual child care quality of all parents, i.e. the planner does not observe q_p . However, in order to fix ideas, let us first consider that the planner observes both w and q_p .

Since the planner perfectly observes (w, q_p) , the policies that the planner will apply may therefore depend on these two characteristics: $w_0(w, q_p)$, $p_f(w, q_p)$, while due to technological and ethical constraints, the other policies, namely q_f and n which characterize the organization of the formal child care supply, apply homogeneously to all households. We show here that when the planner has complete information about the parents, there exists a second best set of policies which achieve the first best, despite parents' freedom of child care choice.

Proposition 3 *The second best under complete information: When parental care quality is public information,*

1. *there exists a policy $P^{SB}(w, q_p) \equiv (w_0^{SB}(w, q_p), p_f^{SB}(w, q_p))$ such that the first best allocation is*

⁸This can be shown more formally by deriving the planner's first order conditions with respect to the 4 instruments and by combining them. Such an exercise indeed reveals that those 4 equations only conveys 2 independent informations.

achieved. In particular, $P^{SB}(w, q_p)$ ensures perfect redistribution among households, with $c(w, q_p) = c^{FB}, \forall (w, q_p)$, and an optimal time use: $t_f^{BR}(w, q_p) = t_f^{FB}(w, q_p)$.

2. This policy is the following:

$$p_f^{SB} = t_f^{FB}w - c^{FB} + (1 - t_f^{FB}) \left(\frac{\gamma}{\gamma + \delta} C'_\tau + \frac{\delta}{\gamma + \delta} w \right), \quad (18)$$

$$w_0^{SB} = c^{FB} - t_f^{FB}w + t_f^{FB} \left(\frac{\gamma}{\gamma + \delta} C'_\tau + \frac{\delta}{\gamma + \delta} w \right). \quad (19)$$

3. The optimal price of formal child care is monotonically increasing in the parent's wage, with

$$\frac{\partial p_f^{SB}}{\partial w} = t_f^{FB} + (1 - t_f^{FB}) \frac{\delta}{\gamma + \delta} \in [0, 1].$$

4. The optimal amount of parental allowance is monotonically decreasing in the parent's wage, with

$$\frac{\partial w_0^{SB}}{\partial w} = -\frac{\gamma}{\gamma + \delta} t_f^{FB} \in [-1, 0].$$

Proof. Provided in Appendix 3. ■

Corollary 1 When parental care quality is public information and when parents perfectly internalize child quality ($\delta = 0$), the optimal policy is given by

$$p_f^{SB}(\delta = 0) = C'_\tau + t_f^{FB}(w - C'_\tau) - c^{FB}, \quad (20)$$

$$w_0^{SB}(\delta = 0) = c^{FB} - t_f^{FB}(w - C'_\tau), \quad (21)$$

with

$$\begin{aligned} \frac{\partial p_f^{SB}}{\partial w} &= t_f^{FB} \in [0, 1], \\ \frac{\partial w_0^{SB}}{\partial w} &= -t_f^{FB} \in [-1, 0]. \end{aligned}$$

The first important message conveyed by Proposition 3 is that the first best allocation can be reached by designing the appropriate child care policy. Technically speaking, thanks to the fact that the household type is observable, child care policies can be individualized, which gives the planner two instruments to reach two objectives for each household. Indeed, at the first best, the planner assigns a level of consumption and of formal child care use to each household. It turns out that the parental allowance w_0 and the price of formal child care p_f have similar incentive effects and opposite effects on income redistribution: On the one hand, an increase in either of them induces a lower use of formal child care. On the other hand, a higher parental allowance increases the consumption level of the beneficiary, while an increase in the price of formal child care has an opposite effect. It is therefore possible to combine both instruments in such a way that the first best allocation is achieved. For instance, inducing a higher use of formal child care for a household whose consumption must be increased can be done by reducing the price. If on the contrary the first best allocation assigns a higher use of formal child care to a household whose consumption must be reduced, the planner can decrease the parental allowance.

Second, we can characterize this optimal policy. Our results indicate that the price of formal child care must be increasing and the parental allowance decreasing in the parent's wage. Recall that the planner pursues the twofold objective of income redistribution and incentive provision.

In order to disentangle the effects of both objectives on the optimal policy, let us discuss the case where parents perfectly internalize child quality ($\delta = 0$). In this case indeed, the planner's and the parent's objective are aligned as far as child quality is concerned. As a result, the planner's choice of policy is only guided by redistributive motives. Corollary 1 tells us that the price remains increasing and the allowance decreasing in the wage. The reason thereof is that high income parents must be taxed for perfect redistribution to be achieved. However, because a high price would reduce high types' labor supply, the planner needs to decrease the parental allowance (which has similar redistribution effects) in order to maintain adequate incentives to work. Moreover, by taking the sum of equations (20) and (21), we see that, under perfect internalization, $p_f^{SB} + w_0^{SB} = C'_\tau$. This simply tells us that the marginal receipt of formal child care must equal its marginal cost. In other words, parents should pay the full cost of formal child care use. Indeed, if the use of formal child care increases by 1 unit of time, the parent pays a price p_f^{SB} but also an opportunity cost of w_0^{SB} , both going (back) to the public budget.

When child quality is imperfectly internalized by parents ($\delta > 0$), the issue of incentive compatibility is slightly more complex. In this respect, the sum of equations (18) and (19) is informative:

$$p_f^{SB}(w) + w_0^{SB}(w) = \frac{\gamma}{\gamma + \delta} C'_\tau + \frac{\delta}{\gamma + \delta} w.$$

We see that the full price paid by the parent, namely the price plus the opportunity cost, is a weighted average of the marginal cost and the parent's wage. As a useful reminder, notice that the first best allocation entails a positive correlation between the wage w and the parental care quality q_p among parents at an interior solution regarding time use (see Proposition 1 and Figure 1).⁹ More precisely, among parents at an interior solution, w is lower than C'_τ if and only if q_p is lower than q_f . We thus note that, contrary to the case of perfect internalization, the full price paid by low types ($q_p < q_f$) is lower than the marginal cost C'_τ , while it is higher for high types. This price scheme aims at providing the right incentives. On the one hand, low types, who underweight child quality and who must be given an incentive to increase their use of formal child care, are subsidized by the planner. On the other hand, for the same reason, high types are taxed in order to induce them to increase their supply of, higher quality, parental care.

Looking more closely at the impact of parent's imperfect internalization of child quality on the shape of child care policies, we see that

$$\begin{aligned} \frac{\partial^2 p_f^{SB}}{\partial \delta \partial w} &= \frac{\gamma}{(\gamma + \delta)^2} (1 - t_f^{FB}) > 0, \\ \frac{\partial^2 w_0^{SB}}{\partial \delta \partial w} &= \frac{\gamma}{(\gamma + \delta)^2} t_f^{FB} > 0. \end{aligned}$$

The former expression states that the price scheme, which is increasing in the parent's wage, is steeper when the planner needs to incentivize parents, that is to say when its extra weight on children δ is higher. The latter expression tells us that the parental allowance, which is decreasing in the parent's wage, is flatter under imperfect internalization.

As a final remark pertaining to the case of complete information, notice that, if the planner were to loose one instrument, a tradeoff would naturally arise between redistribution and incentive provision. For instance, if the price were independent of the wage, then the planner would need to adopt a flatter (and potentially

⁹This is always true, even if this correlation is not present in the whole population.

upward sloping) parental allowance function in order to induce low types to increase their use of formal child care and high types to decrease it. This would prevent the planner from achieving perfect redistribution.

4.3 The second best under incomplete information

Because of the planner's inability to observe q_p , the price and allowance policies that the planner can apply only depend on w : $w_0(w)$, $p_f(w)$.¹⁰ This results in parents with heterogeneous parental care qualities facing the same child care policy.

The planner's objective is to maximize social welfare (3) with respect to the functions $w_0(w)$ and $p_f(w)$, while satisfying the following budget constraint, which takes account of the receipt of formal child care and of the expenses on parental allowance:

$$B = \iint [t_f^* p_f(w) - (1 - t_f^*) w_0(w)] h(w, q_p) dw dq_p + G - F \geq 0, \quad (22)$$

where G is the exogenous and fixed endowment allocated to child care policies and F is the cost of formal child care facilities as defined in equation (8). Let us write the maximization problem in the form of the following Lagrangean:

$$\underset{w_0(w), p_f(w)}{\text{Max}} L = W + \lambda B,$$

where λ is the Lagrange multiplier on the budget constraint and hence the shadow price of public funds. The first order conditions of this problem involve the marginal impacts on welfare and on the budget. Let us study them separately.

In what follows, it is important to remember that we are studying the impact on households with a given wage w . When we highlight and interpret heterogenous effects, we are therefore referring to parents who differ in their quality of parental care q_p .

4.3.1 Impact on welfare

The parental allowance

We start by studying the welfare effects of the parental allowance. The marginal welfare effect of an increase in the parental allowance granted to parents with wage w is given by

$$\frac{dW}{dw_0(w)} = \int \frac{d[U(c, q) + \delta v(q)]}{dw_0(w)} h(w, q_p) dq_p,$$

where

$$\frac{d[U(c, q) + \delta v(q)]}{dw_0} = \frac{\partial U(c, q)}{\partial w_0} + \frac{\partial [U(c, q) + \delta v(q)]}{\partial t_f} \frac{\partial t_f^{BR}}{\partial w_0}.$$

¹⁰One could argue that, based on the observation of the parent's response, namely his/her choice of t_f , the planner should be able to infer q_p . This is conceptually true (see Lemma 1). However, we assume a timing of actions such that the planner does not have the possibility to revise p_f and w_0 once t_f has been chosen, mainly for its lack of realism. Indeed, governments generally commit to policies. In other words, in principle, rules are not modified ex post. Moreover, if the planner were allowed to play again after parents, the latter would strategically integrate this in their decision. It is unclear whether, given these incentives to act strategically, the planner would still be able to retrieve q_p from parents' choices.

More precisely,

$$\begin{aligned}
\frac{d[U(c, q) + \delta v(q)]}{dw_0(w)} &= 0, \text{ if } q_p < \tilde{q}_p^{1, BR}(w) \\
&= u'(c) (1 - t_f^*) + \delta v'(q) (q_f - q_p) \frac{\partial t_f^*}{\partial w_0}, \text{ if } \tilde{q}_p^{1, BR}(w) \leq q_p \leq \tilde{q}_p^{0, BR}(w), \\
&= u'(c), \text{ if } q_p > \tilde{q}_p^{0, BR}(w).
\end{aligned}$$

Notice first that parents at corner solutions do not change their behavior at the margin. Hence, the welfare effect on "corner households" only corresponds to the direct positive effect on their consumption. Parents whose parental care quality is lower than $\tilde{q}_p^{1, BR}(w)$ make full use of formal child care and do not benefit from an increase in the parental allowance. On the contrary, parents with a very high quality of care, namely higher than $\tilde{q}_p^{0, BR}(w)$, and who do not work take full advantage of a higher parental allowance, their utility level increasing by $u'(c)$. As far as "interior households" are concerned, they benefit from a higher consumption level in the proportion of the time spent at home and modify their behavior. More precisely, they decrease their formal child care use ($\partial t_f^*/\partial w_0 < 0$). By the envelope theorem, this change in behavior does not affect their utility: $\partial U(c, q)/\partial t_f = 0$. However, if their internalization of child quality is imperfect ($\delta > 0$), they generate an externality, which amounts to

$$\chi_q(q_p) \equiv \delta v'(q) (q_f - q_p). \quad (23)$$

This term measures the externality in child quality resulting from a marginal increase in the use of formal child care by a parent of type q_p . It is positive for low types and negative for high types.

The price of formal child care

Similarly, let us turn to the welfare effect of an increase in the price of formal child care charged to parents with wage w .

$$\frac{dW}{dp_f(w)} = \int \frac{d[U(c, q) + \delta v(q)]}{dp_f(w)} h(w, q_p) dq_p,$$

where,

$$\begin{aligned}
\frac{d[U(c, q) + \delta v(q)]}{dp_f(w)} &= -u'(c), \text{ if } q_p < \tilde{q}_p^{1, BR}(w) \\
&= -t_f^* u'(c) + \chi_q(q_p) \frac{\partial t_f^*}{\partial p_f(w)}, \text{ if } \tilde{q}_p^{1, BR}(w) \leq q_p \leq \tilde{q}_p^{0, BR}(w), \\
&= 0, \text{ if } q_p > \tilde{q}_p^{0, BR}(w).
\end{aligned}$$

Again, the welfare effect of a policy parameter, here the price of formal child care, depends on the household type. Low quality parents, who make full use of formal child care, are fully hit by the increase in the price. Their utility decreases by $u'(c)$. On the contrary, high quality parents, who rear their child full time, are unaffected by a rise in prices. Finally, "interior parents" are hit in the proportion of their use of formal child care. They also modify their behavior, thereby reducing the externalities they generate (positive for low types and negative for high types).

4.3.2 Impact on the budget

The parental allowance

Taking the derivative of the budget constraint (22) with respect to $w_0(w)$, we find that the budgetary impact of a marginal change in the parental allowance granted to parents with wage w is given by

$$\frac{dB}{dw_0(w)} = \int \left(- (1 - t_f^{BR}) + \chi_b(w) \frac{\partial t_f^{BR}}{\partial w_0} \right) h(w, q_p) dq_p,$$

where,

$$\chi_b(w) \equiv p_f(w) + w_0(w) - C'_\tau. \quad (24)$$

Two effects are at play. The first one is straightforward as it consists of the cost of increasing the allowance in proportion of the time spent at child rearing by type w parents, on average (considering that parents with different care qualities q_p adopt different levels of t_f^{BR}). The second effect relates to the the monetary cost, or benefit, of some parents' change in formal child care use. We denote the budgetary impact of a marginal increase in formal child care use of type w parents by $\chi_b(w)$. This term measures the difference between the full price paid by parents and the marginal cost of formal child care. If it is positive, then parents are taxed. If it is negative, then they are subsidized for their use of formal child care.

The price of formal child care

A similar analysis leads to

$$\frac{dB}{dp_f(w)} = \int \left(t_f^{BR} + \chi_b(w) \frac{\partial t_f^{BR}}{\partial p_f} \right) h(w, q_p) dq_p.$$

A higher price relaxes the budget constraint but also tends to reduce formal child care use. Again, the sign of the latter effect depends on whether type w parents are taxed ($\chi_b(w) > 0$), or subsidized ($\chi_b(w) < 0$).

By combining the impacts on welfare and on the budget, one can write the planner's set of first order conditions with respect to its child care policies as follows

$$\begin{aligned} \frac{dL}{dw_0(w)} &= \int \left[(u'(c) - \lambda) (1 - t_f) + (\chi_b(w) + \chi_q(q)) \frac{\partial t_f^*}{\partial w_0} \right] h(w, q_p) dq_p = 0, \forall w, \\ \frac{dL}{dp_f(w)} &= \int \left[(u'(c) - \lambda) (-t_f) + (\chi_b(w) + \chi_q(q)) \frac{\partial t_f^*}{\partial p_f} \right] h(w, q_p) dq_p = 0, \forall w. \end{aligned}$$

Rearranging and making use of Proposition 2, we obtain the following system of equations, which characterize the optimal child care policy for any level of wage w :

$$\int [(u'(c) - \lambda) + (\chi_b(w) + \chi_q(q)) \Delta_I] h(w, q_p) dq_p = 0, \forall w \quad (25)$$

$$\int [(u'(c) - \lambda) (-t_f) + (\chi_b(w) + \chi_q(q)) (\Delta_S - t_f \Delta_I)] h(w, q_p) dq_p = 0, \forall w. \quad (26)$$

What should be retained from these expressions is that, by playing with the parental allowance and the price of formal child care, the planner triggers two types of effects which influence parents' decisions: the substitution effect Δ_S and the income effect Δ_I . As stated in the following proposition, the presence of an income effect prevents the planner from achieving perfect income redistribution. Indeed, contrary to the complete information case, we show that the objectives of redistribution and incentive provision are, in the general case, conflicting when q_p is private information.

Proposition 4 *The second best under incomplete information:* When parental care quality is private information,

1. *low wage parents are subsidized and high wage parents are taxed for their use of formal child care:*

$$\chi_b(w) < 0 \iff w < C'_\tau,$$

2. *in the presence of an income effect, perfect income redistribution cannot be achieved, neither between wage levels, nor within a given wage level.*

3. *in the absence of an income effect, perfect redistribution can be achieved on average between wage levels:*

$$E_{q_p} [u'(c) | w] = \lambda, \forall w,$$

but inequalities persist among households with different parental care qualities, for any given wage level.

Proof. The proof of point 1 is provided in Appendix 4. Points 2 and 3 are shown and discussed in the following lines. ■

The first important lesson from Proposition 4 is that the incentive scheme highlighted in the case of complete information is qualitatively unchanged: under imperfect internalization of child quality by parents, low types' use of formal child care, which generates positive externalities, must be enhanced by public subsidies $\chi_b(w) < 0$. The converse is true for high types, who must still be encouraged to provide a higher amount of parental care.

Second, we see that income redistribution is challenged by the lack of information and the ensuing lack of instruments for the planner. It can be analyzed at two levels. On the one hand, the planner aims at redistributing income between wage levels. On the other hand, owing to the second dimension of heterogeneity, namely parental care quality, parents with equal wages still make different child care decisions, resulting in different consumption levels. Hence the planner's willingness to redistribute also within a given wage level. The information about income redistribution under the optimal child care policy is carried by condition (25). It is straightforward to see that in the absence of an income effect ($\Delta_I = 0$), we have

$$E_{q_p} [u'(c) | w] = \lambda, \forall w.$$

This means that, on average, parents with different wage levels consume the same. In other words, perfect redistribution can be achieved between different wage levels. On the contrary, in light of condition (25), we see that, as soon as Δ_I is different from zero, this is no true anymore. The intuition behind this result is as follows: In the absence of an income effect, redistribution and incentive provision are orthogonal. In other words, by redistributing from high to low income households, the planner does not affect child care choices. In the presence of an income effect, income redistribution has incentive effects and a tradeoff naturally arises between the two. As a result, the planner adopts the best balance between incentives and redistribution, which is therefore imperfect at the planner's optimum.

Similarly to the preceding subsection, let us now briefly explore the case where the parents fully internalize child quality ($\delta = 0$). Again, in this case, the planner and the parent's objectives are aligned, as far as child quality is concerned. This particular case allows us to provide some insights into the issue of income redistribution under incomplete information.

Proposition 5 *When parental care quality is private information and when parents perfectly internalize child quality ($\delta = 0$),*

1. *low wage parents are subsidized and high wage parents are taxed for their use of formal child care:*

$$\chi_b(w) < 0 \iff w < C'_\tau,$$

2. *perfect income redistribution cannot be achieved, neither between wage levels, with low wage parents consuming on average less than high wage parents:*

$$E_{q_p} [u'(c) | w] > \lambda \iff w < C'_\tau,$$

nor within a given wage level.

Proof. Provided in Appendix 5. ■

One can immediately see that those results are qualitatively equivalent to the case of imperfect internalization of child quality. However, the context and more precisely the objective of the planner is different. Indeed, as mentioned earlier, the planner only aims in this case at redistributing income. This cannot be done perfectly in the general case when an income effect is at play, like under imperfect internalization, but the rationale behind it is different: under incomplete information, parents with equal wages but heterogeneous parental care qualities face the same policy parameters. They however make different choices. In particular, for each wage level, relatively high quality parents have a lower use of formal child care, which gives them a lower (higher) consumption level when $\chi_b(w)$ is negative (positive). We thus see that if $\chi_b(w)$ is upward sloping, then inequalities emerge within wage levels. It must be noted that, at the same time, an upward sloping price scheme $\chi_b(w)$ mitigates inequalities between wage levels, by redistributing income from high wage to low wage parents. There is therefore a tradeoff between within and between inequality reduction. As a result, the second best price scheme does not entail perfect redistribution between wage levels, as stated in point 2 of the proposition.

5 Conclusion

While there is abundant evidence of the effect of formal child care on parents' labor supply and on child development, the goals and forms of child care policies are still under debate. With this paper, we aimed at providing a theoretical contribution to this debate. We have adopted a normative angle on a selection of papers in the existing child care literature. This has allowed us to highlight that formal child care generates externalities, which affect the current and the future generation in several ways. This assessment motivated our assumption that parents may imperfectly internalize the social benefits of child quality. We have considered a population of parents who differ in both their earnings potential and their quality as childminders. Our first best analysis provides insights into the goals of child care intervention, while the second best approach allows us to characterize the optimal design of child care policies.

In the first best analysis, we have identified and discussed the tradeoff between labor market participation and child quality, which arises differently according to the parent's type. Under the assumption of a uniform level of formal child care quality, we have described the optimal organization of the supply side in terms of the quality and the size of child care facilities.

We have analyzed the shape of parents' reaction to labor taxes and child care policy instruments. This allowed us to show that, if the planner can manipulate both the price scheme and the parental allowance,

then fiscal instruments are redundant, which is consistent with Domeij and Klein (2013)'s results on the substitutability of child care subsidies to reductions in labor taxes.

Not surprisingly, an adequate use of child care policies allows the planner to decentralize the first best allocation. Our analysis of optimal policies reveals that redistributive motives lead to a price and a parental allowance, which are respectively upward and downward sloping in the parent's wage. When the planner needs to incentivize parents in order to enhance child quality, the price scheme becomes steeper and the parental allowance function flatter.

Under incomplete information, our results indicate that redistribution and the production of child quality becomes conflicting objectives. We still get that low wage parents are subsidized for their use of formal child care, while high income parents are taxed. However, it becomes impossible to reach perfect redistribution both within and between wage levels.

An important limitation of the present paper consists of its over-simplified representation of the supply side. Indeed, in reality, the childcare market is characterized by a complex heterogeneous supply composed of public, private and informal providers competing in prices and quality. It would be interesting to check whether our results are robust to the introduction of public - private competition. Besides and more fundamentally, this complexity of the supply side would deserve a normative analysis per se. Indeed, one would like to know whether we can find a rationale for the observed heterogeneity in the quality of child care services and for the coexistence of public and private providers.

6 References

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7 Appendices

7.1 Appendix 1: Proof of Proposition 1

The objective of the planner can be written in the form of the following Lagrangean:

$$\underset{c(w, q_p), t_f(w, q_p), q_f, n}{Max} \quad L = W_p + \rho W_k + \lambda B,$$

where W_p , W_k and B are respectively given by equations (5), (6) and (7) and where λ is the Lagrange multiplier of the budget constraint. We successively analyze the different first order conditions of the planner’s optimization problem.

Let us start with the allocation of private consumption among parents:

$$\begin{aligned} \frac{\partial L}{\partial c(w, q_p)} &= 0 \iff u'(c(w, q_p)) = \lambda, \forall (w, q_p) \\ &\implies c(w, q_p) = c(w', q'_p), \forall (w, q_p), (w', q'_p). \end{aligned}$$

Second, the first order condition with respect to time use of formal child care is as follows:

$$\frac{\partial L}{\partial t_f(w, q_p)} = [\rho v'(q(w, q_p))(q_f - q_p) + \lambda(w - C'_\tau)] h(w, q_p).$$

Hence

$$t_f^{FB} = 0 \iff \frac{\partial L(t_f = 0)}{\partial t_f(w, q_p)} \leq 0 \iff \rho v'(q_p)(q_f - q_p) + \lambda(w - C'_\tau) \leq 0, \quad (27)$$

$$t_f^{FB} = t_f^{**}(w, q_p) \in (0, 1) \iff \frac{\partial L}{\partial t_f(w, q_p)} = 0 \iff \rho v'(q(t_f^{**}, q_p))(q_f - q_p) + \lambda(w - C'_\tau) = 0, \quad (28)$$

$$t_f^{FB} = 1 \iff \frac{\partial L(t_f = 1)}{\partial t_f(w, q_p)} \geq 0 \iff \rho v'(q_f)(q_f - q_p) + \lambda(w - C'_\tau) \geq 0. \quad (29)$$

Notice that the second order condition is satisfied, which ensures that our problem is well-behaved:

$$\frac{\partial^2 L}{\partial t_f(w, q_p)^2} = \left[\rho v''(q(w, q_p))(q_f - q_p)^2 h(w, q_p) - \lambda n C''_\tau \cdot \left(\frac{1}{n} h(w, q_p) \right)^2 \right] h(w, q_p) < 0.$$

The two frontiers between the corner and interior solutions to the planner's optimization problem are written $\tilde{q}_p^{0,FB}$ and $\tilde{q}_p^{1,FB}$ and are respectively such that conditions (27) and (29) are satisfied with equality. In order to draw figure 1, which illustrates the mapping from the household type to formal child care time use in the space (w, q_p) , we need to study these frontiers:

The frontier between $t_f^{FB} = 0$ and $t_f^{FB} = t_f^{**} > 0$, which we denote by $\tilde{q}_p^{0,FB}(w)$ is implicitly given by

$$\rho v'(\tilde{q}_p^{0,FB})(q_f - \tilde{q}_p^{0,FB}) = \lambda(C'_\tau - w). \quad (30)$$

Making use of the implicit function theorem, we obtain the slope of this frontier:

$$\begin{aligned} \frac{dq_p}{dw} \Big|_{t_f^{FB}=0} &= - \frac{\lambda}{\rho v''(\tilde{q}_p^{0,FB})(q_f - \tilde{q}_p^{0,FB}) - \rho v'(\tilde{q}_p^{0,FB})} > 0 \\ &\iff \frac{v''(\tilde{q}_p^{0,FB})}{v'(\tilde{q}_p^{0,FB})} (q_f - \tilde{q}_p^{0,FB}) < 1. \end{aligned} \quad (31)$$

Two cases need to be distinguished. On the one hand, for low types ($q_p < q_f$), this condition is satisfied since the left hand side is clearly negative, by $v''(q_p) < 0$. On the other hand, for high types ($q_p > q_f$), the condition can be rewritten as

$$\frac{dq_p}{dw} \Big|_{t_f^{FB}=0} > 0 \iff - \frac{v''(\tilde{q}_p^{0,FB})}{v'(\tilde{q}_p^{0,FB})} < \frac{1}{\tilde{q}_p^{0,FB} - q_f} \iff - \frac{v''(\tilde{q}_p^{0,FB})}{v'(\tilde{q}_p^{0,FB})} \leq \frac{1}{q_p^H},$$

which is satisfied, by assumption. The second derivative with respect to q_p gives us the concavity:

$$\frac{d^2 q_p}{dw^2} \Big|_{t_f^{FB}=0} = \frac{d^2 q_p}{dq_p dw} \Big|_{t_f^{FB}=0} \frac{dq_p}{dw} \Big|_{t_f^{FB}=0},$$

where $\frac{dq_p}{dw} \Big|_{t_f^{FB}=0} > 0$ and where

$$\frac{d^2 q_p}{dq_p dw} \Big|_{t_f^{FB}=0} = \lambda \rho \frac{v''(\tilde{q}_p^{0,FB})(q_f - \tilde{q}_p^{0,FB}) - 2v''(\tilde{q}_p^{0,FB})}{\left[\rho v''(\tilde{q}_p^{0,FB})(q_f - \tilde{q}_p^{0,FB}) - \rho v'(\tilde{q}_p^{0,FB}) \right]^2} > 0,$$

since $v'''(q) \geq 0$. Therefore $\frac{d^2 q_p}{dw^2} \Big|_{t_f^{FB}=0} > 0$.

The frontier between $t_f^{FB} = t_f^{**} < 1$ and $t_f^{FB} = 1$, which we denote by $\tilde{q}_p^{1,FB}$, is given by

$$\rho v'(q_f)(q_f - \tilde{q}_p^{1,FB}) = \lambda(C'_\tau - w). \quad (32)$$

Isolating $\tilde{q}_p^{1,FB}$, we find the following linear relationship:

$$\tilde{q}_p^{1,FB}(w) = q_f + \frac{\lambda}{\rho v'(q_f)} (w - C'_\tau). \quad (33)$$

Therefore, $\left. \frac{dq_p}{dw} \right|_{t_f^{FB}=1} > 0$ and $\left. \frac{d^2 q_p}{dq_p dw} \right|_{t_f^{FB}=1} = 0$.

Also, we need to show that t_f^{**} is increasing in w and decreasing in q_p . To this end, let us apply the implicit function theorem to equation (28). We obtain

$$\begin{aligned} \frac{\partial t^{**}}{\partial w} &= -\frac{\lambda}{\rho v''(q(w, q_p)) (q_f - q_p)^2 h(w, q_p) - \lambda n C''_\tau \cdot \left(\frac{1}{n} h(w, q_p)\right)^2} > 0, \\ \frac{\partial t^{**}}{\partial q_p} &= -\rho \frac{v''(q) t_f (q_f - q_p) - v'(q)}{\rho v''(q(w, q_p)) (q_f - q_p)^2 h(w, q_p) - \lambda n C''_\tau \cdot \left(\frac{1}{n} h(w, q_p)\right)^2} < 0 \\ &\iff v''(q) t_f (q_f - q_p) - v'(q) < 0. \end{aligned}$$

On the one hand, for low types ($q_p < q_f$), this condition is satisfied. On the other hand, for high types ($q_p > q_f$), the condition can be rewritten as

$$\frac{\partial t^{**}}{\partial q_p} < 0 \iff -\frac{v''(q)}{v'(q)} < \frac{1}{t_f (q_p - q_f)} \iff -\frac{v''(q)}{v'(q)} \leq \frac{1}{q_p^H},$$

by assumption.

Finally, let us mention two additional useful elements that help us drawing figure 1.

First, we know that

$$\tilde{q}_p^{0,FB}(C'_\tau) = \tilde{q}_p^{1,FB}(C'_\tau) = q_f,$$

by (30) and (32).

Second, we observe that both frontiers are equally sloped at $(w, q_p) = (C'_\tau, q_f)$. Indeed, by (31) and (33), we have

$$\left. \frac{dq_p(C'_\tau)}{dw} \right|_{t_f^{FB}=0} = \left. \frac{dq_p(C'_\tau)}{dw} \right|_{t_f^{FB}=1} = \frac{\lambda}{\rho v'(q_f)}.$$

Turning to the organization of the sector of formal child care, we now identify the first best level of quality and the optimal number of child care facilities.

Regarding quality q_f , the first order condition is as follows:

$$\frac{\partial L}{\partial q_f} = 0 \iff \rho \iint t_f(w, q_p) v'(q) h(w, q_p) dq_p dw = \lambda n C'_q.$$

Regarding the number of child care facilities n ,

$$\frac{\partial L}{\partial n} = -\lambda \frac{\partial B}{\partial n} = 0 \iff \kappa + C(\tau_f, q_f) - n C'_\tau(\tau_f, q_f) \frac{T_f}{n^2} = 0.$$

7.2 Appendix 2: Proof of Proposition 2

For interior parents, the first order condition with respect to time use is satisfied with equality:

$$\frac{\partial U}{\partial t_f} = u'(c) [(1 - \theta) w - p_f - w_0] + \gamma v'(q(t_f^*, q_p)) (q_f - q_p) = 0. \quad (34)$$

Applying the implicit function theorem to this equation allows to find readily the expressions appearing in points 1 to 4 of Proposition 2. Let us now study the sign of these expressions.

Reaction to the lump sum transfer:

In light of expression (16), we have that

$$\frac{\partial t_f^*}{\partial r} = \Delta_I < 0 \iff (1 - \theta)w - p_f - w_0 > 0.$$

Because only interior parents react at the margin, we can make use of the first order condition (34), which tells us that

$$(1 - \theta)w - p_f - w_0 > 0 \iff q_p > q_f.$$

Reaction to the labor tax:

In light of expressions (16) and (17), we have

$$\begin{aligned} \frac{\partial t_f^*}{\partial \theta} &= w(\Delta_S - t_f \Delta_I) < 0 \\ \iff u''(c) t_f^* ((1 - \theta)w - p_f - w_0) + u'(c) &> 0. \end{aligned}$$

Making use of the first order condition (34), we know that low types (those for whom $q_p < q_f$) have $(1 - \theta)w - p_f - w_0 < 0$. Therefore, for them $dt_f^*/d\theta < 0$. On the contrary, high types have $(1 - \theta)w - p_f - w_0 > 0$. For high types, the condition can be rewritten as

$$\frac{\partial t_f^*}{\partial \theta} < 0 \iff -\frac{u''(c)}{u'(c)} < \frac{1}{t_f^* ((1 - \theta)w - p_f - w_0)} \iff -\frac{u''(c)}{u'(c)} \leq \frac{1}{w^H},$$

which is satisfied, by assumption.

Reaction to the parental allowance:

Making use of (16) and (17), we have

$$\begin{aligned} \frac{\partial t_f^*}{\partial w_0} &= (1 - t_f) \Delta_I + \Delta_S < 0 \\ \iff -u''(c) (1 - t_f^*) ((1 - \theta)w - p_f - w_0) + u'(c) &> 0. \end{aligned}$$

For high types, $(1 - \theta)w - p_f - w_0 > 0$ and the condition is automatically satisfied. For low types, the condition can be rewritten as

$$\begin{aligned} \frac{\partial t_f^*}{\partial w_0} < 0 &\iff -\frac{u''(c)}{u'(c)} < \left| \frac{1}{(1 - t_f^*) ((1 - \theta)w - p_f - w_0)} \right| \\ \iff -\frac{u''(c)}{u'(c)} < \left| \frac{1}{((1 - \theta)w - p_f - w_0)} \right| &\iff -\frac{u''(c)}{u'(c)} \leq \frac{1}{w^H}. \end{aligned}$$

Indeed, low types have $(1 - \theta)w - p_f - w_0 < 0$. The case where the constraint is the most likely to be binding is hence $w = 0$. The condition is then satisfied for $p_f + w_0 < w^H$. As it is unlikely that the full cost of formal child care (the price plus the opportunity cost w_0) be larger than the highest wage in the economy in equilibrium, we have $\partial t_f^*/\partial w_0 < 0$ for low types as well.

Reaction to the price of formal child care:

In light of expressions (16) and (17), we have

$$\begin{aligned} \frac{\partial t_f^*}{\partial p_f} &= \Delta_S - t_f \Delta_I < 0 \\ \iff u''(c) t_f^* ((1 - \theta)w - p_f - w_0) + u'(c) &> 0, \end{aligned}$$

which as be shown above (see the proof on parent's reaction to the labor tax).

7.3 Appendix 3: Proof of Proposition 3

Given that the full household type is observable, the planner has at its disposal 2 instruments per household, namely w_0 and p_f . For any given household, the First Best allocation is determined by a system of 2 equations, which determine a consumption and a time use target for the planner:

$$c(w, q_p; w_0, p_f) = (1 - t_f^{BR}) w_0 + t_f^{BR} (w - p_f) = c^{FB}, \quad (35)$$

$$t_f^{BR}(w, q_p; w_0, p_f) = t_f^{FB}(w, q_p). \quad (36)$$

where c^{FB} and t_f^{FB} are defined by (9) and (10), respectively and where t_f^{BR} is given by Lemma 1. We show that there exists a unique solution $(w_0^{SB}(w, q_p), p_f^{SB}(w, q_p))$ to this system of equations:

Let us first determine the level of parental allowance w_0 that allows to reach the First Best consumption level for any given price of formal child care. Rearranging equation (35), we find

$$w_0^{SB}(p_f) = \frac{c^{FB} - t_f^{BR}(w_0^{SB}, p_f)(w - p_f)}{1 - t_f^{BR}(w_0^{SB}, p_f)}. \quad (37)$$

In a second step, let us reproduce here the condition that determines parent's best response to child care policies, which is given by Lemma 1:

$$\gamma v'(q(t_f^*, q_p))(q_f - q_p) = -u'(c)(w - p_f - w_0),$$

given that $\theta = 0$. Let us evaluate parent's reaction at $w_0 = w_0^{SB}(p_f)$, the level of parental allowance at which the consumption level is equal to c^{FB} , such that $u'(c^{FB}) = \lambda$ (see Proposition 1). Substituting, we find

$$v'(q(t_f^*, q_p))(q_f - q_p) = -\lambda \frac{w - p_f - w_0}{\gamma}.$$

The condition for First Best time use is as follows (see Proposition 1):

$$v'(q(t_f^{**}, q_p))(q_f - q_p) = -\lambda \frac{w - C'_\tau}{\rho}.$$

Equalizing the right hand sides of both conditions gives us the price scheme that leads the parent to make a socially optimal decision:

$$t_f^* = t_f^{**} \iff p_f^{SB} + w_0^{SB} = \frac{\gamma}{\gamma + \delta} C'_\tau + \frac{\delta}{\gamma + \delta} w, \quad (38)$$

since $\rho = \gamma + \delta$.

We have a system of 2 equations ((37) and (38)) and 2 unknowns, namely p_f^{SB} and w_0^{SB} . We solve this system by substitution and end up with expressions (18) and (19) in Proposition 3.

Finally, points 3 and 4 of the proposition result from simple derivatives of equations (18) and (19).

7.4 Appendix 4: Proof of Proposition 4

We tackle here the proof of point 1, which states that parents are subsidized $\chi_b(w) < 0$ if and only if their wage is lower than the marginal cost of formal child care $w < C'_\tau$. We start from the system composed of

equations (25) and (26) and study the term $\int (u'(c) - \lambda) t_f h(w, q_p) dq_p$, which appears in equation (26): given that

$$h(w, q_p) = h_{q_p|w}(q_p | w) h_w(w),$$

where $h_{q_p|w}$ and h_w respectively denote the conditional density of q_p and the marginal density of w . From the definition of the covariance, we can write

$$\int (u'(c) - \lambda) t_f h(w, q_p) dq_p = h_w(w) Cov[(u'(c) - \lambda), t_f | w] + E_{q_p}[t_f | w] \int (u'(c) - \lambda) h(w, q_p) dq_p.$$

From equation (25), we have that

$$\int (u'(c) - \lambda) h(w, q_p) dq_p = - \int (\chi_b(w) + \chi_q(q)) \Delta_I h(w, q_p) dq_p.$$

Substituting, we can write

$$\begin{aligned} \int (u'(c) - \lambda) t_f h(w, q_p) dq_p &= h_w(w) Cov[(u'(c) - \lambda), t_f | w] - \chi_b(w) E_{q_p}[t_f | w] \int \Delta_I h(w, q_p) dq_p \\ &\quad - E_{q_p}[t_f | w] \int \chi_q(q) \Delta_I h(w, q_p) dq_p, \end{aligned} \quad (39)$$

because $\chi_b(w)$ does not depend on q_p (see equation (24)). Substituting into equation (26) and isolating $\chi_b(w)$, we find

$$\chi_b(w) = \frac{h_w(w) Cov[(u'(c) - \lambda), t_f | w] - \int \chi_q(q_p) (\Delta_S + (E_{q_p}[t_f | w] - t_f) \Delta_I) h(w, q_p) dq_p}{\int (\Delta_S + (E_{q_p}[t_f | w] - t_f) \Delta_I) h(w, q_p) dq_p}. \quad (40)$$

Let us now study the sign of the terms appearing in this expression.

From Lemma 1, we know that low types, for whom $q_p < q_f$, necessarily have $w < p_f + w_0$ if they are at an interior solution with respect to time allocation. Therefore for them $\partial c / \partial t_f < 0$, as can be seen from the expression of consumption (13). Because $u''(c) < 0$, $u'(c)$ is then an increasing function of t_f . As a result, the covariance between $u'(c) - \lambda$ and t_f is positive for low types and negative for high types. We now focus on the term $\Delta_S + (E_{q_p}[t_f | w] - t_f) \Delta_I$, which appears both in the numerator and the denominator. Making use of the expressions of Δ_I (16) and Δ_S (17), one obtains

$$\Delta_S + (E_{q_p}[t_f | w] - t_f) \Delta_I < 0 \iff -u''(c) (E_{q_p}[t_f | w] - t_f) (w - p_f - w_0) + u'(c) > 0,$$

given that we are studying the case where $\theta = 0$. By Lemma 1, $w - p_f - w_0$ is negative for low types and positive for high types. Besides, conditional on w , t_f will be higher than average for (relatively) low quality parents and lower than average for (relatively) high quality parents.¹¹ Therefore, the condition is always satisfied for relatively low quality parents among the low types ($t_f > E_{q_p}[t_f | w]$ and $q_p < q_f$). This is also true for relatively high quality parents among the high types ($t_f < E_{q_p}[t_f | w]$ and $q_p > q_f$). For the other 2 categories, the condition can be rewritten as

$$\Delta_S + (E_{q_p}[t_f | w] - t_f) \Delta_I < 0 \iff -\frac{u''(c)}{u'(c)} < \left| \frac{1}{(E_{q_p}[t_f | w] - t_f) (w - p_f - w_0)} \right|.$$

¹¹Notice that in equilibrium, high quality parents are still low types ($q_p < q_f$) when $w < C'_\tau$, whereas low quality parents are still high types ($q_p > q_f$) when $w < C'_\tau$. This is because we are conditioning on w .

Making use of the fact that $|E_{q_p}[t_f | w] - t_f| \in [0, 1]$, we see that $\Delta_S + (E_{q_p}[t_f | w] - t_f) \Delta_I$ is negative under the sufficient condition that

$$-\frac{u''(c)}{u'(c)} < \left| \frac{1}{w - p_f - w_0} \right|,$$

which always holds under our assumption that $-u''(c)/u'(c) \leq 1/w^H$. The conclusion of this analysis is that $\Delta_S + (E_{q_p}[t_f | w] - t_f) \Delta_I$ is always negative.

The final step is to remind that $\chi_q(q_p)$ is positive for low types and negative for high types (see equation (23)).

Coming back to the expression under study (40), we can conclude that the denominator is negative. Regarding the numerator, it follows from the above analysis that it is positive for low types and negative for high types, which completes the proof that χ_b is negative for low types ($w < C'_\tau$) and positive for high types ($w > C'_\tau$).

7.5 Appendix 5: Proof of Proposition 5

In the case of perfect internalization of child quality, we have that $\delta = 0$ and hence $\chi_q = 0$, by equation (23). Accordingly we can rewrite the conditions for optimal child care policy ((25) and (26)) as

$$\int [(u'(c) - \lambda) + \chi_b(w) \Delta_I] h(w, q_p) dq_p = 0, \forall w, \quad (41)$$

$$\int [(u'(c) - \lambda)(-t_f) + \chi_b(w) (\Delta_S - t_f \Delta_I)] h(w, q_p) dq_p = 0, \forall w. \quad (42)$$

First, making use of equation (39), where we take account of the fact that $\chi_q = 0$, we can rewrite the latter condition (42) as

$$\chi_b(w) = \frac{h_w(w) Cov[(u'(c) - \lambda), t_f | w]}{\int (\Delta_S + (E_{q_p}[t_f | w] - t_f) \Delta_I) h(w, q_p) dq_p}.$$

We show in Appendix 4 that, on the one hand, the numerator of this expression is positive for low types and negative for high types. On the other hand, the denominator is shown to be negative for all type. It follows that $\chi_b(w)$ is negative for low types, for whom $w < C'_\tau$, and positive for high types, for whom $w > C'_\tau$, as stated in point 1 of the Proposition.

Second, rearranging equation (41), we get

$$\int (u'(c) - \lambda) h(w, q_p) dq_p = -\chi_b(w) \int \Delta_I h(w, q_p) dq_p.$$

Combining Proposition 2 (Δ_I is positive for low types) and the first point of the present Proposition ($\chi_b(w)$ is negative for low types), we obtain that

$$\int (u'(c) - \lambda) h(w, q_p) dq_p > 0 \iff w < C'_\tau,$$

which shows point 2 of the Proposition.

